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Reliability analysis of gear-bearing drive systems considering gear manufacturing and installation errors

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Gear-bearing drive systems often exhibit manufacturing and installation errors, which can significantly affect system performance, and longevity, and increase the probability of failures. This paper focuses on the reliability analysis of gear-bearing drive systems with uncertainties in system parameters such as gear backlash and bearing clearance, caused by gear and bearing manufacturing and installation errors. First, a dynamic model of the gear-bearing drive system, incorporating coupled dynamic meshing parameters, is established. Then, the deterministic dynamic model of the system is combined with the Chebyshev interval analysis method to develop a reliability analysis model for the gear-bearing drive system with uncertain parameters. The study analyzes the variations in system natural frequencies and vibration responses due to gear quality and initial gear and bearing clearances at different deviation rates. The results indicate that at the same rotational speed and deviation rate, the initial bearing clearance has a more significant impact on the system's dynamic characteristics compared to the initial gear clearance. At different rotational speeds and the same deviation rate, system reliability decreases with increasing average initial interference of the bearing at low speeds. At high speeds, a large bearing clearance deviation may cause abnormal fluctuations in system vibration. This method provides a prioritization of parameter control for the structural optimization and design of gear-bearing systems.

Keywords Gear-bearing drive system, Uncertain parameters, Interval algorithm, Backlash uncertainty, Dynamic meshing parameters

The power systems of aircraft[1–3], vehicles[4], wind turbines[5], and ships[6] are all complex rotating mechanical systems that involve numerous uncertain internal and external factors. With the increasing performance demands on rotating machinery, there are higher requirements for the manufacturing precision and assembly clearances of transmission components such as gears and bearings[7].

The dynamic analysis of gear-bearing transmission systems inherently involves internal parameter uncertainties arising from limitations in manufacturing precision and unavoidable assembly errors. These uncertainties, particularly those associated with gear backlash and bearing clearance, directly govern system performance and operational reliability. Consequently, investigating the dynamic characteristics of such systems under uncertain gear backlash and bearing clearance conditions holds critical importance for understanding nonlinear vibration mechanisms, predicting failure modes, and optimizing tolerance design to mitigate performance degradation.

At present, a substantial body of research has been conducted by numerous scholars on the uncertainty issues in gear transmission systems. According to the employed methodologies, the models can generally be categorized into three types: probabilistic models, fuzzy models, and interval models. Among them, probabilistic models, which are based on probability theory and statistics, are widely applied in cases involving material parameter fluctuations and random vibrations due to their well-established theoretical framework and clear probabilistic interpretation[8–10]. Yu[11] employed a PC-Kriging adaptive algorithm to analyze the reliability of thermo-structural-dynamic coupled systems, addressing the high-temperature failure issues in aero-engine gear-rotor systems. Hajnayeb et al.[12] proposed the use of power spectral density and frequency response functions to study the

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effects of various random manufacturing errors on the vibration response of bearings. Feng et al.[13] adopted an extended interval stochastic method to analyze the interval natural frequencies of systems with mixed uncertain parameters. Onur Can Kalay[14] et al. proposed a one-dimensional convolutional neural network (1-D CNN) model for gear systems with cracks and tooth asymmetry. Probabilistic models have also been widely used to investigate uncertainties in wind turbine gear systems[15–18]. However, stochastic models typically require a large amount of experimental data for support. Fuzzy models[19], based on fuzzy mathematical analysis, are another type of uncertainty quantification method. They offer advantages such as not relying on probability distributions and exhibiting strong flexibility. Jing[20] employed a genetic fuzzy immune PID algorithm to tune immune parameters for controlling system responses. Zhao[21] applied a fuzzy comprehensive evaluation method to determine the optimal combination of processing parameters for non-circular gears. Gu[22] developed an integrated gear fault diagnosis model by combining a Hidden Markov Model (HMM) with a fuzzy evaluation model. However, fuzzy models are generally not well-suited for systems involving multiple uncertain parameters. Both probabilistic and fuzzy models tend to have high computational complexity. In scenarios where data are scarce[23] or rapid evaluation of transmission system robustness is required[24], interval models are more suitable.

Interval models, which are characterized by low information requirements, high computational efficiency, and strong robustness assurance, have been widely applied by researchers in the uncertainty analysis of gear systems. Wu[25] was the first to combine interval analysis with multibody dynamics, proposing an interval algorithm based on Chebyshev polynomial function approximation. Due to its rapid convergence and compatibility with various dynamic models, this interval method has since been applied by scholars to efficiently evaluate uncertainty in dynamic engineering problems. Subsequently, Wei[26–28] was the first to introduce the Chebyshev interval analysis method into gear dynamics, demonstrating its feasibility through both numerical simulations and experimental validation. The study revealed that even small variations in certain uncertain parameters within a limited range can lead to significant uncertainty in system responses. Hu[29] proposed a multi-degree-of-freedom nonlinear dynamic model of a spur gear system with misalignment uncertainty, based on Chebyshev polynomial function approximation. Zhao[30] combined interval and stochastic analysis to develop a hybrid interval uncertainty structural dynamic analysis method. Guerine, A[31] introduced a method based on polynomial chaos projection to evaluate the uncertainty in the dynamic response of gear systems. Guo[32] proposed a method based on polynomial chaos expansion (PCE) to analyze the uncertainty in gear manufacturing errors. Beyaoui, M[33] developed a computational approach to assess the robustness of wind turbine gearbox system responses while considering uncertainties in wind direction and blade pitch angle. Wu[34] used PCE to model the uncertainties in mesh stiffness, bearing stiffness, and damping parameters of a double-helical gear–bearing system. Wei et al.[35] studied the dynamic response of gear transmission systems with uncertainties in mass, mesh stiffness, and support stiffness using an interval analysis method based on Chebyshev inclusion functions. Yuhang Hu et al.[29] also adopted the Chebyshev inclusion function approach to analyze misalignment uncertainties in multi-degree-of-freedom gear systems. Chao-Fu[36] combined PCE with polynomial surrogate analysis (PSA) to address hybrid aleatory and epistemic uncertainties in transmission systems. Bel-Mabrouk[37] proposed a polynomial chaos-based approach to study aerodynamic parameter uncertainties in bevel gear systems. Najib[38] investigated the effect of static transmission error uncertainty, caused by gear manufacturing deviations, on the dynamic response of gear systems. In summary, previous studies have mainly focused on the application of interval analysis methods to parameter uncertainties in gear systems. However, the uncertainty associated with gear backlash and bearing clearance in gear–bearing transmission systems has not yet been adequately addressed.

The objective of this study is to propose a reliability analysis method for gear–bearing transmission systems considering gear manufacturing and installation errors. By integrating the Chebyshev interval analysis method with a gear transmission system dynamic model that incorporates coupled dynamic meshing parameters, the interval estimation of the inherent characteristics and vibration responses of a gear–bearing transmission system is investigated under uncertainties in gear mass, initial gear backlash, and initial bearing clearance. The proposed method is applicable for evaluating the dynamic behavior of gear transmission systems when uncertainties in gear backlash and bearing clearance are present.

The motivation of this paper is to propose a reliability analysis method for gear-bearing transmission systems that accounts for gear manufacturing and installation errors. By integrating the Chebyshev interval analysis method with a dynamic model of the gear transmission system that incorporates coupled dynamic meshing parameters, the paper achieves interval estimation of the inherent characteristics and vibration responses of the gear-bearing transmission system under uncertainties in gear quality, initial gear clearance, and initial bearing clearance. This method is suitable for evaluating the interval variations in dynamic characteristics of gear transmission systems when the ranges of certain uncertain parameters are known.

The remainder of this paper is organized as follows. In the second section, a dynamic model of the gear-bearing drive system has been established, considering dynamic meshing parameters. Then, in the third section, a modeling method for gear-bearing drive systems based on the Chebyshev interval analysis method has been developed, and the relevant formulas have been derived. In the fourth section, the uncertainty of the system's natural frequencies under uncertain mass parameters has been calculated and analyzed. Finally, in the fifth section, the variation ranges of gear meshing parameters, gear backlash, and bearing clearance have been analyzed under different initial gear and bearing clearances.

Dynamic modeling of gear-bearing systems

In practical engineering, factors such as manufacturing precision and assembly errors lead to uncertainties in the initial gear backlash, gear mass, and initial bearing clearance within gear-bearing drive systems. However, these parameters are constrained within specific ranges according to design guidelines. Due to the presence of numerous nonlinear factors in the system, even slight variations in parameters can have a significant impact on the system. Therefore, these uncertainties are essential considerations in the dynamic modeling of gear-bearing drive systems. To more clearly analyze the dynamic characteristics of gear-bearing drive systems, this paper introduces a reliability analysis modeling method for gear-bearing drive systems based on the Chebyshev interval analysis method.

First, a dynamic model with deterministic parameters is established for the gear-bearing transmission system, focusing on the coupled relative positions of gears, dynamic gear backlash, and dynamic bearing clearance. It is assumed that the gear system moves only within a plane, and all gears are treated as rigid discs[39]. Changes in the relative positions of the gears affect the meshing force.

The variation in the geometric positional relationship of the gear-bearing transmission system at adjacent moments is shown in Fig. 1. At the previous moment, the mass centers are denoted as $G_1$and G_2, and the geometric centers of the system are denoted as O_1 and O_2. At the subsequent moment, the geometric centers change to $C_1\,(x_p, y_p)C_1\,(x_p, y_p)$ and $C_2\,(x_g, y_g)$. At this point, the relative position of the gears changes, resulting in alterations in the center distance L, deflection angle β, and engagement angle α.

$$L = \sqrt{(L_0 + x_g - x_p)^2 + (y_g - y_p)^2} \tag{1}$$

$$\beta = \tan^{-1}\left[(y_g - y_p)/(L_0 + x_g - x_p)\right] \tag{2}$$

$$\alpha = \cos^{-1}(r_{bp} + r_{bg})/L \tag{3}$$

where L_0 represents the original center distance, r_b represents the base circle radius, and r_a represents the addendum circle radius. Subscripts p and g denote the driving and driven gears, respectively.

Each gear can be represented by three generalized coordinates, x, y and θ_z. Considering factors such as gravity and torque, the equations of motion for the gear-bearing system are established and expressed as Eq. (4).

$$M^s\ddot{q}^s + C^s\dot{q}^s = F_w + F_m + F_{Tg} - F_b^s \tag{4}$$

The mass matrix is expressed asM^s.

$$M^s = \begin{bmatrix} m_p & 0 & 0 & 0 & 0 & 0 \\ 0 & m_p & 0 & 0 & 0 & 0 \\ 0 & 0 & J_p & 0 & 0 & 0 \\ 0 & 0 & 0 & m_g & 0 & 0 \\ 0 & 0 & 0 & 0 & m_g & 0 \\ 0 & 0 & 0 & 0 & 0 & J_g \end{bmatrix} \tag{5}$$

where m_p, J_p, m_g, and J_g represent the masses and moments of inertia of the input and output gears, respectively. The damping matrix $C^s$is denoted as Eq. (6).

$$C^s = diag\,(c_{x_1}, c_{y_1}, 0, c_{x_2}, c_{y_2}, 0) \tag{6}$$

The terms F_b^s, F_{Tg}, F_w, and F_m on the right-hand side of Eq. (4) are expressed as Eq. (7) through (10), respectively.

$$F_b^s = diag\,(F_{bx_1}, F_{by_1}, 0, F_{bx_2}, F_{by_2}, 0) \tag{7}$$

$$F_{Tg} = \begin{bmatrix} 0 & -m_p g & T_p & 0 & -m_g g & -T_g \end{bmatrix}^{\mathrm{T}} \tag{8}$$

$$F_w = \begin{bmatrix} m_p\rho_p\,\dot{\theta}_{z_p}^2 \cos\varphi_p & m_p\rho_p\theta_{z_p}^2 \sin\varphi_p & 0 & m_g\rho_g\theta_{z_g}^2 \cos\varphi_g & -m_g\rho_g\theta_{z_g}^2 \sin\varphi_g & 0 \end{bmatrix}^{\mathrm{T}} \tag{9}$$

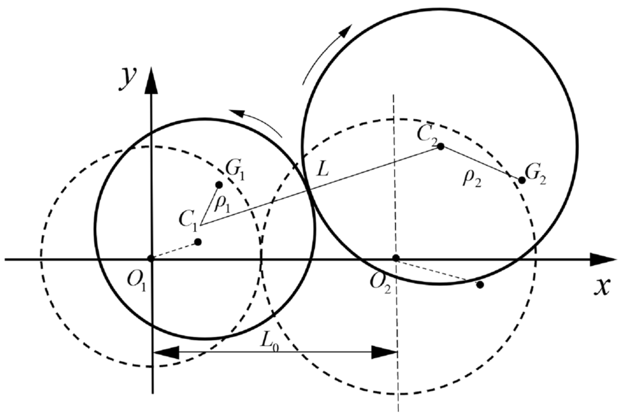


Fig. 1. Schematic diagram of the positional relationship of the system at previous and subsequent moments.

$$F_m = -{F_d}^m \left[\begin{array}{cccccc} \frac{\partial {F_d}^m}{\partial x_p} & \frac{\partial {F_d}^m}{\partial y_p} & \frac{\partial {F_d}^m}{\partial \theta_{zp}} & \frac{\partial {F_d}^m}{\partial x_g} & \frac{\partial {F_d}^m}{\partial y_g} & \frac{\partial {F_d}^m}{\partial \theta_{zg}} \end{array} \right]^{\mathrm{T}} \tag{10}$$

Among them, the gear meshing force on the tooth surface can be expressed by Eq. (11). Ball bearings are used to support the gear system and are simplified into a bearing model with time-varying clearance. The specific expression is given in Eq. (12).

$$ {F_d}^m = F_{md}^k + F_{md}^c = \begin{cases} k_m^d \left[\delta_d - b(t)\right] + c_m^d \left[\dot{\delta}_d - \dot{b}(t)\right] & \delta_d > b(t) \\ 0 & |\delta_d| \leqslant b(t) \\ k_m^d \left[\delta_d + b(t)\right] + c_m^d \left[\dot{\delta}_d + \dot{b}(t)\right] & \delta_d \leqslant -b(t) \end{cases} \tag{11}$$

$$\left[\begin{array}{c} F_{bx_i} \\ F_{by_i} \end{array} \right]\Bigg|_{i=1,2} = K_{bear} \sum_{k=1}^{N_b} \delta_k^{3/2} H\left(x_i \cos\theta_k + y_i \sin\theta_k - \delta_0\right) \left[\begin{array}{c} \cos\theta_k \\ \sin\theta_k \end{array} \right], (k = 1,\ 2,\ \cdots,\ N_{\mathrm{b}}) \tag{12}$$

In Eq. (11), k_m^d represents the time-varying meshing stiffness of the gear, calculated using the potential energy method[40,41] during coupled gear vibration. The calculation method of gear meshing stiffness based on dynamic meshing parameters can be derived from Eq. (1) to (3) in combination with the potential energy method. The damping ratio c_m^d, dynamic transmission error δ_d, and half tooth side clearance $b(t)$ are represented by Equations (13) through (15), respectively. Where ξ_{m} is the damping coefficient, J_p and J_g represent the moments of inertia of the driving and driven gears, respectively, e_a denotes the static transmission error magnitude, ω_p is the angular velocity of the driving gear, and b_0 is the initial half tooth side clearance. In Eq. (12), $K_{bear}$is the Hertz contact stiffness of each ball. $H(\cdot)$serves as the criterion for deter-mining the contact between each ball and the bearing. $H(x) = 1\,(x \geqslant 0)$indicates that the kth roller is engaged with the raceway, and $H(x) = 0\,(x < 0)$ indicates that the kth roller is disengaged from the raceway. δ_0 represents the initial bearing clearance, and N_{b} denotes the number of rolling elements.

$$\delta_d = (x_p - x_g)\sin(\alpha - \beta) + (y_p - y_g)\cos(\alpha - \beta) + r_{bp}\theta_{zp} - r_{bg}\theta_{zg} - e_a \sin(\omega_p Z_p t) \tag{13}$$

$$c_m^d = 2\xi_{\mathrm{m}} \sqrt{k_m^d J_p J_g \left(J_g r_{bp}^2 + J_p R r_{bg}^2\right)} \tag{14}$$

$$b(t) = b_0 + (r_{bp} + r_{bg})\left[(\tan(\alpha) - \alpha) - (\tan(\alpha_0) - \alpha_0)\right] \tag{15}$$

Reliability analysis model of the gear-bearing transmission system

For cognitive uncertainties such as manufacturing and installation defects, conducting extensive experiments[42] or statistical analysis is both time-consuming and labor-intensive. This paper employs the Chebyshev interval analysis method, which is computationally convenient and applicable to various dynamic models[43]. These uncertainties can be described using vectors$\vec{\kappa} = (\kappa_1, \kappa_2, \kappa_3, \ldots, \kappa_m)$.

$$\kappa_i = \left[\underline{\kappa_i}, \overline{\kappa_i}\right] = \left\{ \kappa_i \middle| \underline{\kappa_i} \leqslant \kappa_i \leqslant \overline{\kappa_i} \right\}, i = 1, \cdot\cdot\cdot, m \tag{16}$$

The objective function of the gear system can be represented using Chebyshev polynomial fitting.

$$f(\kappa) \approx p_k(\kappa) = \theta_0 + \sum_{i_1=0}^{k} \cdots \sum_{i_m=0}^{k} \theta_{i_1 \ldots i_m} \kappa_1^{i_1} \cdots \kappa_m^{i_m} = Q\varphi \tag{17}$$

$$\theta_{i_1, i_2, \ldots i_k} = \left(\frac{2}{m}\right)^k \sum_{j_1=1}^{m} \ldots \sum_{j_k=1}^{m} f\left(\cos\varphi_{j_1}, \ldots, \cos\varphi_{j_k}\right) \cos i_1\varphi_{j_1} \ldots \cos i_1 \varphi_{j_k} \tag{18}$$

$$\varphi_i = \arccos\left(\frac{2\kappa_i - \left(\underline{\kappa_i} + \overline{\kappa_i}\right)}{\overline{\kappa_i} - \underline{\kappa_i}}\right) = [0, \pi] \tag{19}$$

Here, ψ represents the rearranged Chebyshev polynomial coefficient matrix, Q is the matrix constructed from the values of interpolation points corresponding to n uncertain parameters, and p_i contains the function values at the interpolation points.

$$\psi = \left(Q^T Q\right)^{-1} Q^T P \tag{20}$$

where,

$$
Q=\begin{bmatrix} 1 & \zeta_1(\kappa_1) & \cdots & \zeta_p(\kappa_1) \\ \vdots & \vdots & & \vdots \\ 1 & \zeta_1(\kappa_n) & \cdots & \zeta_p(\kappa_n) \end{bmatrix}_{n*\frac{(m+k)!}{m!k!}} \tag{21}
$$

$$
P=[p_1,p_2,\ldots,p_n]^T \tag{22}
$$

$$
\zeta_p\left(\vec{\kappa}\right)=\left(\kappa_1^k,\kappa_1^{k-1}\kappa_2,\kappa_1^{k-1}\kappa_3,\cdots,\kappa_m^k\right)^T \tag{23}
$$

Since Eq. (19) has explicit upper and lower bounds, the boundaries of Equation can be approximately equivalent to Eqs. (24) and (25).

$$
\underline{f}(\kappa)=f_{(0,\cdots,0)}-\sum\nolimits_{i_1=0}^{k}\sum\nolimits_{i_i=0}^{k}|f(\kappa_1^{(i_1)},\cdots,\kappa_r^{(i_r)})| \tag{24}
$$

$$
\bar{f}(\kappa)=f_{(0,\cdots,0)}+\sum\nolimits_{i_1=0}^{k}\sum\nolimits_{i_i=0}^{k}|f(\kappa_1^{(i_1)},\cdots,\kappa_r^{(i_r)})| \tag{25}
$$

To systematically evaluate the reliability of gear-bearing transmission systems under manufacturing and installation uncertainties, this section proposes a hybrid analytical framework integrating deterministic dynamic modeling with Chebyshev interval analysis. The methodology shown in Fig. 2 follows a rigorous four-phase workflow:

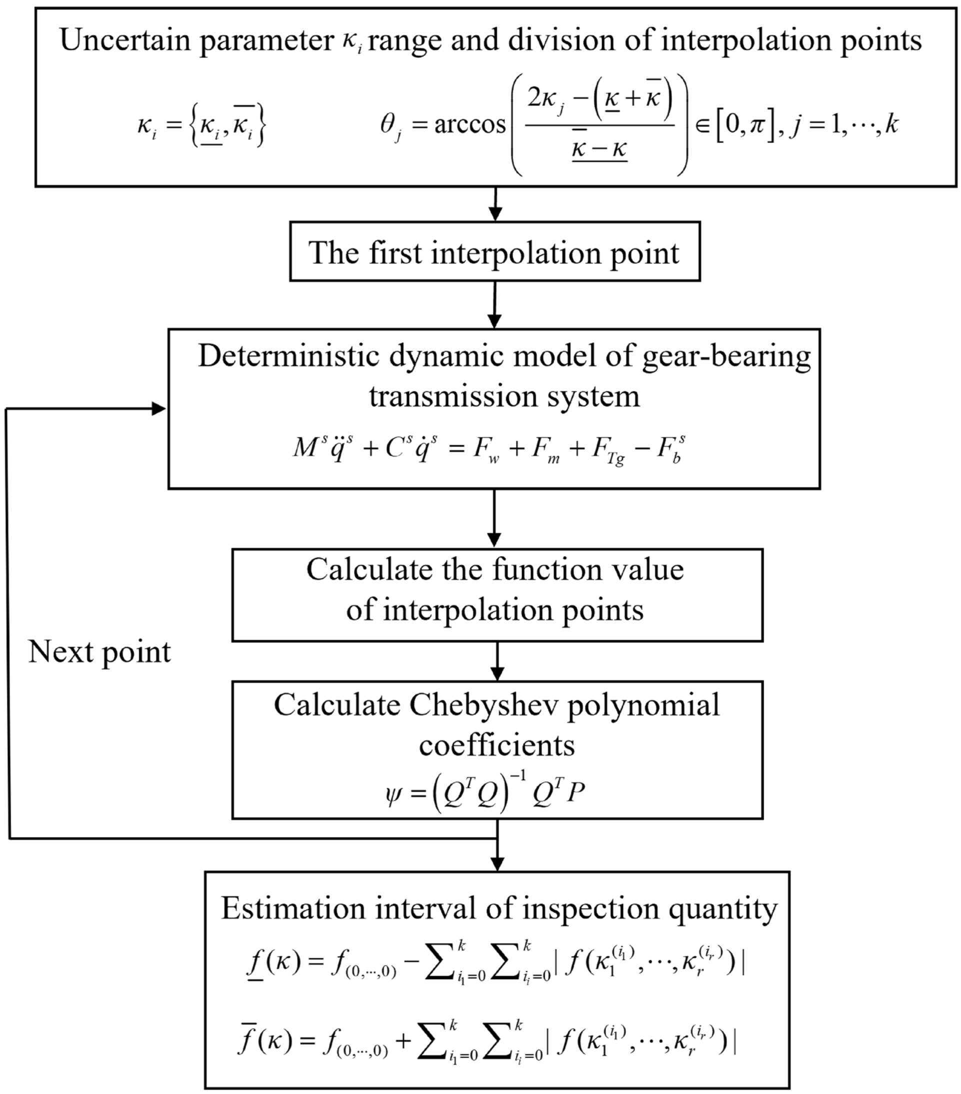


Fig. 2. Flowchart for the analysis of gear-bearing transmission systems based on the Chebyshev interval analysis method.

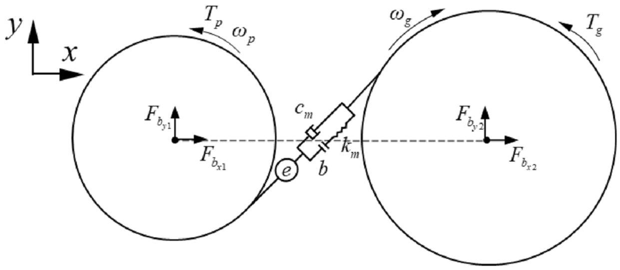


Fig. 3. Schematic diagram of the gear-bearing transmission system structure.

Physical parameters	Variable	Value
Number of teeth of wheel	Z_p,Z_g	20
Modulus (*mm*)	m	30
Elastic modulus (*Gpa*)	E	206
Standard pressure angle (°)	α_0	20
Tooth width (*mm*)	B	22
Standard center distance (*mm*)	L_0	200
Static transmission error (*μm*)	e_a	20
Mass of wheel(*kg*)	m_p,m_g	6.57
Moment of inertia ($kg * m^2$)	J_p,J_g	0.0365
Damping factor	ξ_{m}	0.07
Torque average(*N/m*)	T_{o1},T_{o2}	300
Torque amplitude(*N/m*)	T_{a1},T_{a2}	100
Initial half tooth clearance measurement (*μm*)	b_0	50
Gravitational acceleration (N/m^2)	g	9.85
Standard contact ratio	m_p	1.557
Inner circle radius (*mm*)	r_{i}	10
Outer circle radius (*mm*)	r_o	23.5
Bearing widths (*mm*)	B	14
Number of balls	N_b	10
Bearing damping ($N * s/m$)	c_b	512.64
Hertz contact stiffness (N/m)	K_b	2*10^8

Table 1. Parameters of the Gear-Bearing system Model.

(1) Deterministic System Characterization.

A dynamic model of the gear-bearing transmission system is established, as detailed in Sect. 2, incorporating critical deterministic parameters such as the geometric configurations of gear pairs and bearings, material properties, and operational load conditions.

(2) Uncertainty Quantification.

Key stochastic parameters are identified through manufacturing tolerance analysis, including variations in gear mass, ranges of initial gear backlash, and tolerance intervals of initial bearing clearance. Each parameter is quantified with its respective variation domain.

(3) Chebyshev Surrogate Modeling.

Interval analysis is conducted by evaluating the system's dynamic response using the Chebyshev interval method. This approach enables the numerical determination of bounds for system characteristics, such as natural frequency intervals, vibration response envelopes, and critical dynamic thresholds.

(4) Reliability analysis.

The reliability of the system is assessed by evaluating the extent to which different uncertain parameters influence its dynamic behavior. This analysis identifies priority control parameters that necessitate stringent tolerance management during manufacturing and installation processes, such as backlash-sensitive gear components, mass-critical gear elements, and clearance-dependent bearing assemblies.

The gear-bearing transmission system can be simplified into a schematic of a single-stage gear transmission system, as shown in Fig. 3. Assuming fluctuating input and output torques for the gears, the single-stage gear system is equivalent to rigid disks connected by a time-varying stiffness spring and a time-varying damper. Both the driving and driven gears are considered as lumped mass elements. The specific model parameters of the system are listed in Table 1.

Analysis of system natural frequencies with parameter uncertainty

This section focuses on the impact of mass uncertainty caused by gear manufacturing errors on the natural frequencies of the system. It is assumed that the center of mass is the geometric center of the gear. For the gear-bearing transmission system with parameters listed in Table 1, the first five natural frequencies of the system with deterministic parameters are calculated and presented in Table 2.

Figure 4 shows the fluctuation curves of the first, third, and fifth-order natural frequencies of the system with respect to the deviation rate when the driving gear mass parameter is considered uncertain. It also depicts the fluctuation curves of their upper and lower bounds with respect to the deviation rate. From Figs. 4 (a), (c), and (e), it can be observed that these natural frequencies exhibit a positive correlation with the deviation rate of the uncertain parameter. Figures 4 (b), (d), and (f) reveal that, under the same deviation coefficient, the fifth-order natural frequency f_5 is most sensitive to the variation of the parameter, followed by the third-order natural frequency f_3. Additionally, the fluctuation of the upper and lower bounds of the fifth-order natural frequency is highly symmetric.

Analysis of system vibration response under parameter uncertainty

It is well known that controlling gear clearance and bearing clearance during the design, manufacturing, and installation of gears is crucial to ensure the stability and reliability of the gear-bearing transmission system during operation. Assuming the bearing clearance is based on the clearance between the rolling balls and the bearing outer ring, where the clearance is zero when they are in contact, and ignoring the deformation of the bearing; the variation in gear clearance is described by changes in the half-tooth side clearance. This section investigates the impact of clearance uncertainty on system response by varying the initial bearing clearance and initial gear clearance.

Analysis of system vibration response with uncertainty in initial gear clearance

In this subsection, to investigate the impact of the deviation rate of gear clearance on the reliability of the gear-bearing transmission system, it is assumed that the driving gear speed and initial gear clearance are given as $r = 1000\ rad/min$ and $b_0 = 50\mu m$, respectively. The deviation rates are categorized into seven groups, with the maximum upper and lower limits being 20%. The time-varying states of meshing parameters such as center distance, pressure angle, and deflection angle, as well as the interval ranges of the meshing parameters, are shown on the left and right sides of Fig. 5. The time-varying states of gear clearance and bearing clearance, along with their interval ranges with deviation rates, are shown on the left and right sides of Fig. 6, respectively.

From the meshing parameter variation curves and interval fluctuation ranges shown in Fig. 5, it can be observed that: when the initial gear clearance varies from 0 to 20%, the changes in meshing parameters such as center distance, pressure angle, and deflection angle are relatively small. However, their average fluctuation amplitudes are within the upper and lower boundary ranges.

From the curves of gear and bearing clearances over time and their average value ranges shown in Fig. 6, the following observations can be made: In Figure (a), it is evident that the gear clearance exhibits noticeable variations with different initial gear clearances. Figure (c) describes the change in clearance between a particular rolling ball and the bearing outer ring, showing that the bearing clearance gradually increases from zero to its maximum value and then decreases back to zero. This reflects the rolling ball moving away from and then approaching the bearing outer ring until they make contact, with the entire process exhibiting fluctuating behavior. Figures (b) and (d) illustrate that the impact on gear clearance is significant and increases linearly.

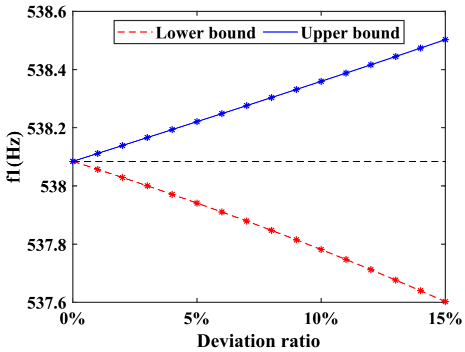
Analysis of system vibration response with uncertainty in initial bearing clearance

In this subsection, to investigate the impact of the deviation rate of the bearing initial clearance on the reliability of the gear-bearing transmission system, it is assumed that the midpoint value of the bearing initial clearance is 30 μm. Two cases of driving gear speeds are considered: 1000 *rad/min* and 4000 *rad/min*. Seven sets of variations are made, with the upper and lower deviation rates having a maximum limit of 20%. The time-varying states of the gear-bearing system parameters such as center distance, pressure angle, and deflection angle, as well as the interval ranges of the meshing parameters, are shown in the left and right sides of Fig. 7. The time-varying states of the gear clearance and bearing clearance and their interval ranges with the deviation rates are shown in the left and right sides of Fig. 8, respectively.

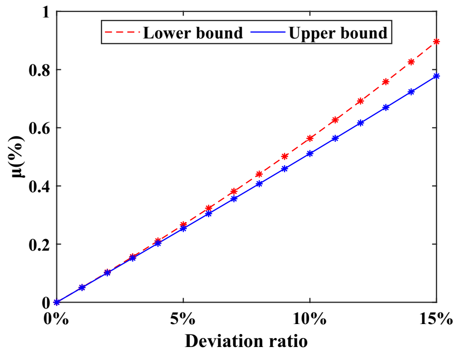
From the curves and interval fluctuation ranges of meshing parameters shown in Fig. 7, it can be observed that: as the initial gear clearance varies from 0 to 20%, the changes in meshing parameters such as center distance, pressure angle, and deflection angle are relatively large. Additionally, the average value fluctuation ranges show a linear increase.

Physical parameters	Variable	Value (Hz)
First-order	f_1	537.63
Second-order	f_2	620.92
Third-order	f_3	2223.98
Fourth-order	f_4	2776.85
Fifth-order	f_5	3634.98

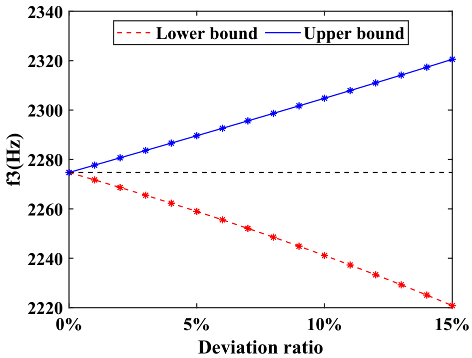
Table 2. Natural frequencies of the transmission system.



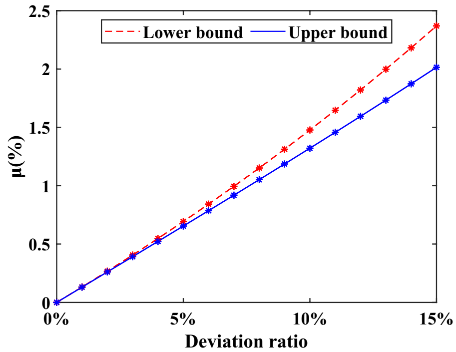
(a) Fluctuation curve of f_1 with respect to the deviation rate



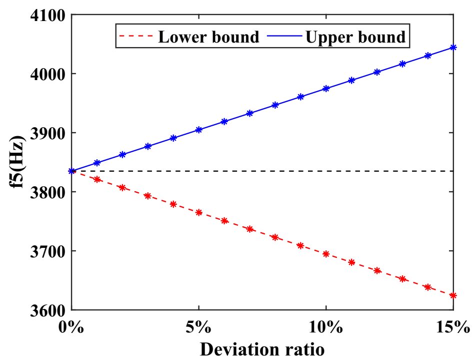
(b) Fluctuation curve of the interval range of f_1 with respect to the deviation rate



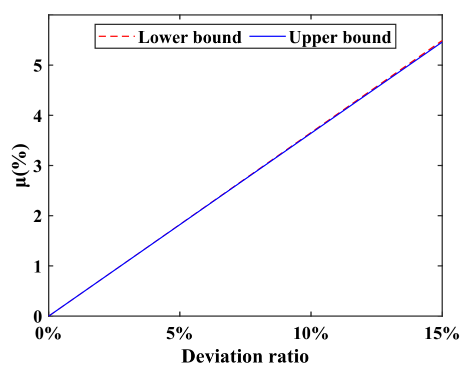
(c) Fluctuation curve of f_3 with respect to the deviation rate



(d) Fluctuation curve of the interval range of f_3 with respect to the deviation rate



(e) Fluctuation curve of f_5 with respect to the deviation rate

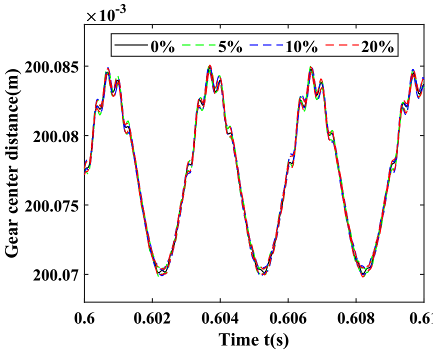


(f) Fluctuation curve of the interval range of f_5 with respect to the deviation rate

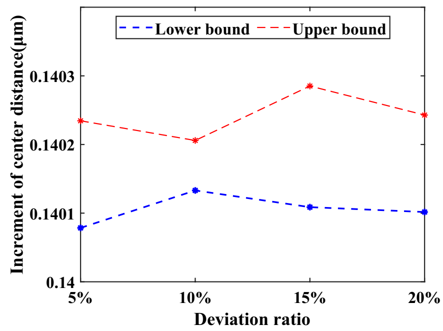
Fig. 4. When m_1 is an interval uncertainty parameter, the curves of natural frequency variation with respect to the deviation coefficient and the corresponding interval fluctuation range.

From Fig. 8, the time-varying curves and average value ranges of gear and bearing clearances reveal the following: In Figure (a), it is evident that the gear clearance shows noticeable changes with different initial gear clearances. Figure (c) describes the variation in the clearance between a ball and the bearing outer ring, showing that the bearing clearance increases from zero to a maximum and then decreases back to zero, indicating the ball moving progressively away from and then towards the bearing outer ring until contact is made, with the entire process exhibiting fluctuations. Figures (b) and (d) illustrate that the bearing clearance increases from 71 μm to 76 μm and the gear clearance increases from 83.5 μm to 84.5 μm, with the bearing clearance showing a more significant increase.

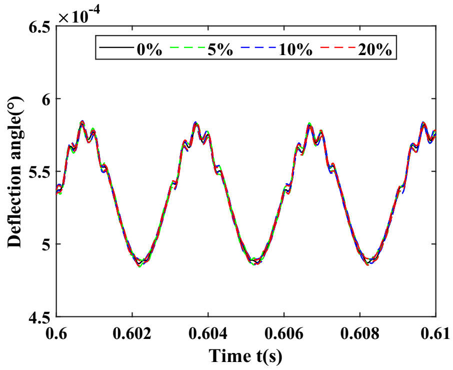
In Figs. 9 and 10, the variation in the time-varying states and average values of gear meshing parameters, as well as gear and bearing clearances, are illustrated for a gear rotational speed of 4000 rad/min, showing how they change with different deviation rates.



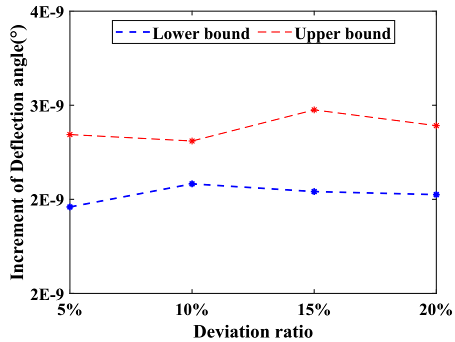
(a) Fluctuation curve of center distance



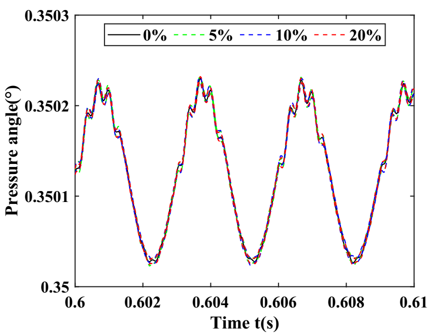
(b) Range of mean amplitude variation of venter distance



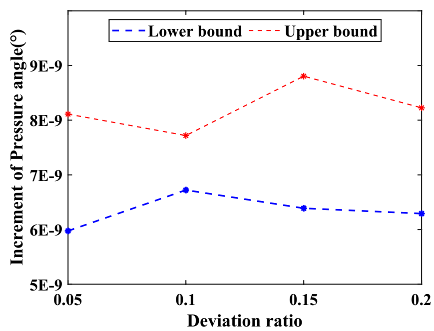
(c) Fluctuation curve of deflection angle



(d) Range of variation in the average value of the deflection angle



(e) Fluctuation curve of the pressure angle



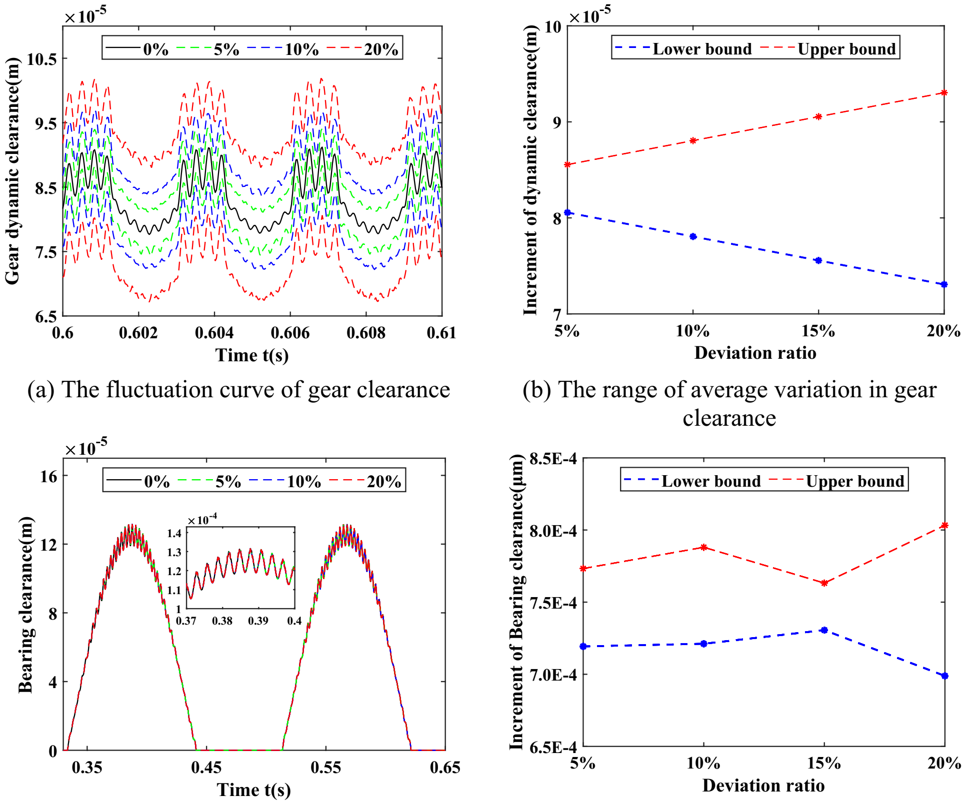
(f) Range of variation in the average value of the pressure angle

Fig. 5. Variation curves and average value ranges of gear meshing parameters with deviation coefficient when b_0 is an interval uncertainty parameter.

From the variation curves and range of fluctuations of the meshing parameters shown in Fig. 9, it can be observed that when the gear initial clearance varies within the range of 0–15%, the trends of parameters such as center distance, pressure angle, and deflection angle are similar. However, there are significant differences when the deviation rate reaches 20%. The average values of these parameters all show a linear increase. Compared to gear clearance, the impact of bearing initial clearance on the system is more pronounced under the same deviation rate conditions.

From Fig. 10, the time-varying curves and average value ranges of gear clearance and bearing clearance clearly show that: In Figures (a) and (c), it is evident that both gear clearance and bearing clearance exhibit significant changes with different deviation rates. The trend of bearing clearance remains consistent with previous observations. Figures (b) and (d) illustrate that bearing clearance increases from 71.2 μm to 76.1 μm and gear clearance increases from 83.9 μm to 84.9 μm, both showing a linear increase. Compared to the trends observed at 1000 rad/min, there are notable changes in the fluctuation trends of gear meshing parameters and gear clearance, with both gear and bearing clearances showing increased average values.

To validate the effectiveness and practicality of the proposed method, we performed a comparative analysis using the model parameters and results reported in References[39]. Figure 11(a) compares the displacement of the driving wheel obtained from our dynamic model with that from the literature. Figure 11(b) shows the theoretical



(a) The fluctuation curve of gear clearance

(b) The range of average variation in gear clearance

(c) Variation curve of bearing clearance

(d) Variation range of the average bearing clearance

Fig. 6. Variation curves and interval fluctuation ranges of gear and bearing clearances with uncertainty parameter b_0.

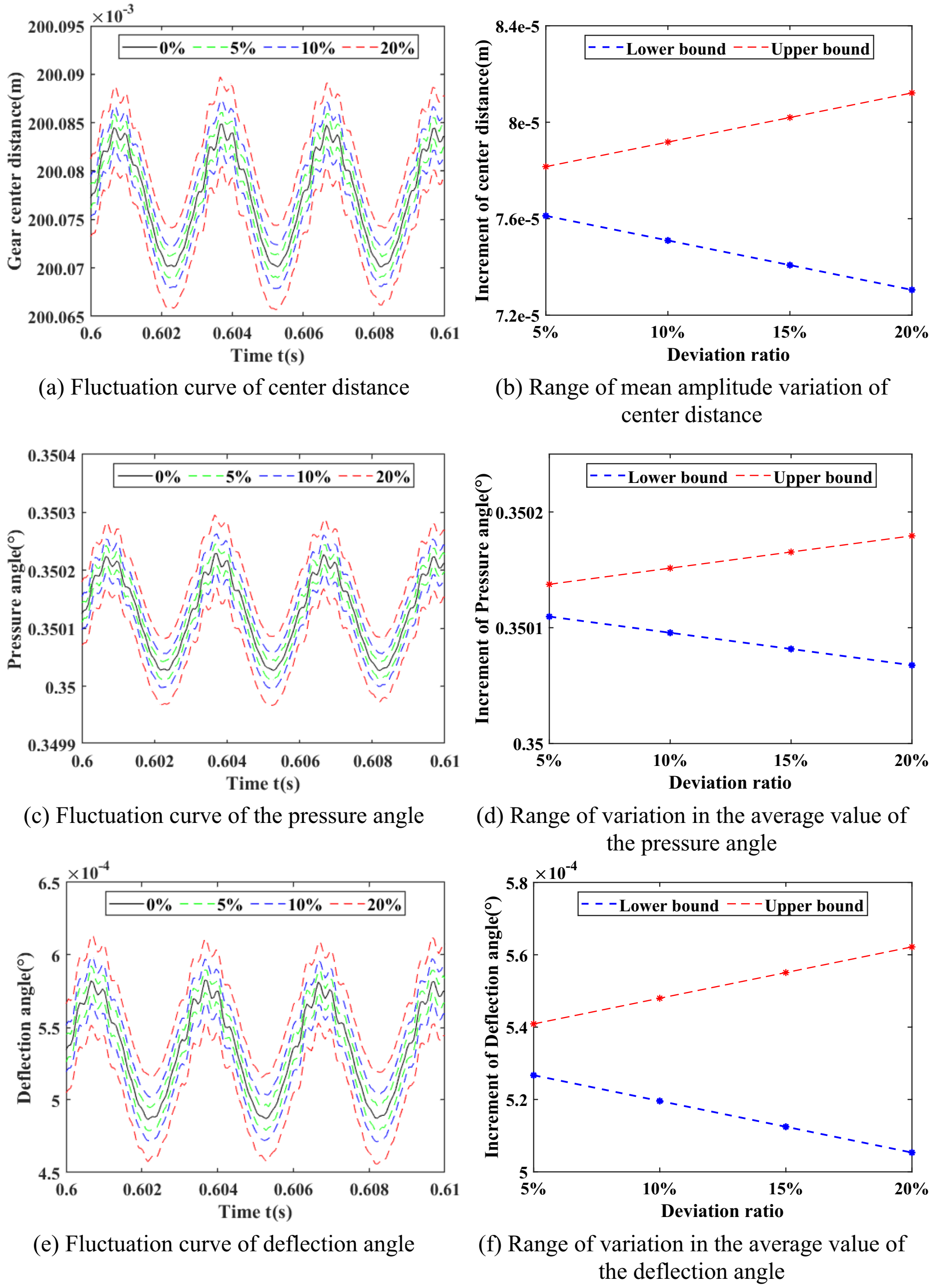
response alongside the literature results for gear backlash varying within 50 μm ± 20%, and Fig. 11(c) presents a close-up of the region highlighted in Fig. 11(b). As shown in Fig. 11, our computed results closely match those reported in the literature. Moreover, the uncertainty analysis indicates that variations in gear backlash have a minimal effect on system behavior, and all literature values fall within our calculated interval bounds.

Conclusions

This paper has proposed a reliability analysis model for gear–bearing transmission systems based on Chebyshev interval analysis methods, aiming to reveal the impact characteristics of manufacturing and installation errors on the dynamic properties of gear systems. First, the dynamic model of the uncertain gear–bearing system has been formulated as a differential equation with uncertain parameters. Next, Chebyshev interval analysis has been incorporated, and numerical integration methods have been used to solve for the target function values. The effects of uncertain gear mass, initial gear backlash, and initial bearing clearance on system reliability have been studied. The main conclusions are as follows:

1. When the mass of the driving wheel is an uncertain parameter, the fifth-order natural frequency is most sensitive to fluctuations under the same deviation rate, and the upper and lower bounds of the fifth-order natural frequency exhibit highly symmetrical fluctuation patterns.
2. Under identical rotational speeds and deviation ratios, the initial bearing clearance demonstrates greater influence on the vibrational characteristics of the proposed gear–bearing transmission system compared to the initial gear backlash.
3. Significant variations emerge in the temporal response patterns of the system under identical bearing initial clearance deviation ratios at different rotational speeds. At specified low-speed conditions, dynamic reliability decreases with increasing initial mean clearance deviations. However, nonlinear interactions arising from clearance-dependent contact transitions (engagement/disengagement states) induce anomalous vibration oscillations at high-speed operations with excessive bearing initial clearance deviation ratios.

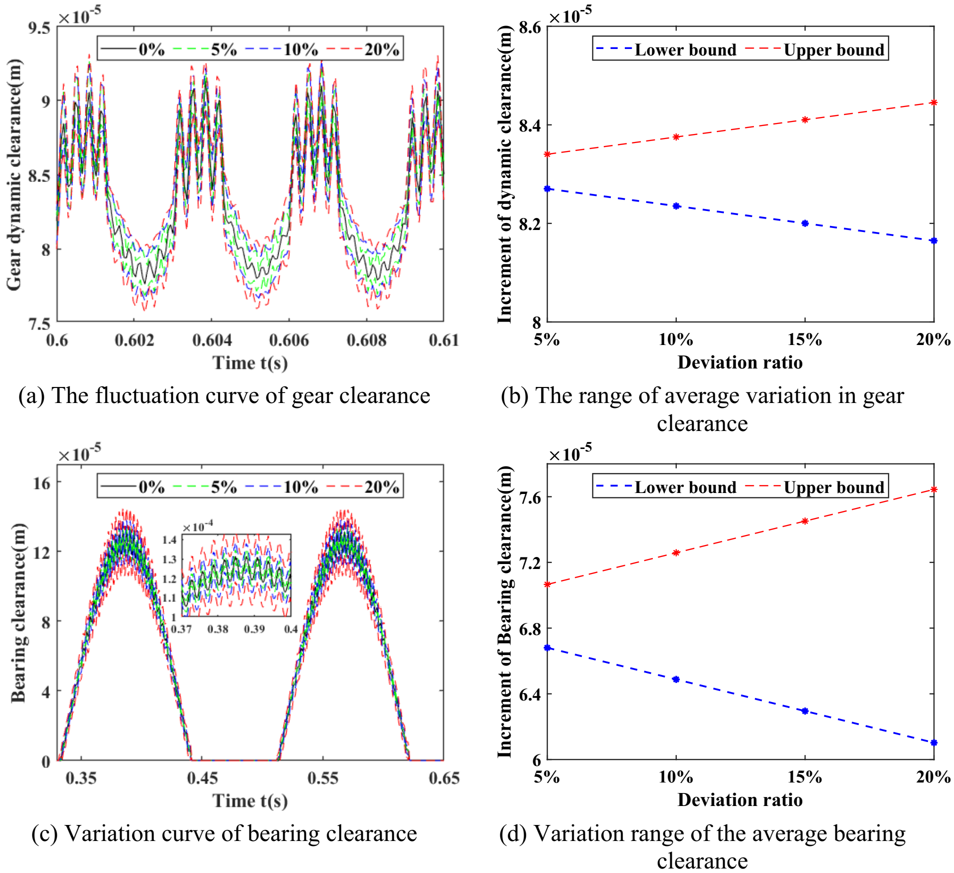
This systematic approach bridges theoretical modeling with practical engineering applications. The results of this analysis can be used to optimize the design and maintenance of gear–bearing systems by considering uncertainties such as gear manufacturing errors and bearing clearance. In practice, the findings can guide engineers in determining optimal tolerance allocations, improving the reliability of gear–bearing systems,



(a) Fluctuation curve of center distance
(b) Range of mean amplitude variation of center distance
(c) Fluctuation curve of the pressure angle
(d) Range of variation in the average value of the pressure angle
(e) Fluctuation curve of deflection angle
(f) Range of variation in the average value of the deflection angle

Fig. 7. Variation of gear meshing parameters with δ_0 at 1000 rad/min for the driving gear.

and minimizing the risk of performance degradation or failure under uncertain operational conditions. By integrating this model into the design phase, manufacturers can ensure better performance and longevity of gear systems, particularly in applications where precision and reliability are critical, such as in automotive, aerospace, and industrial machinery.



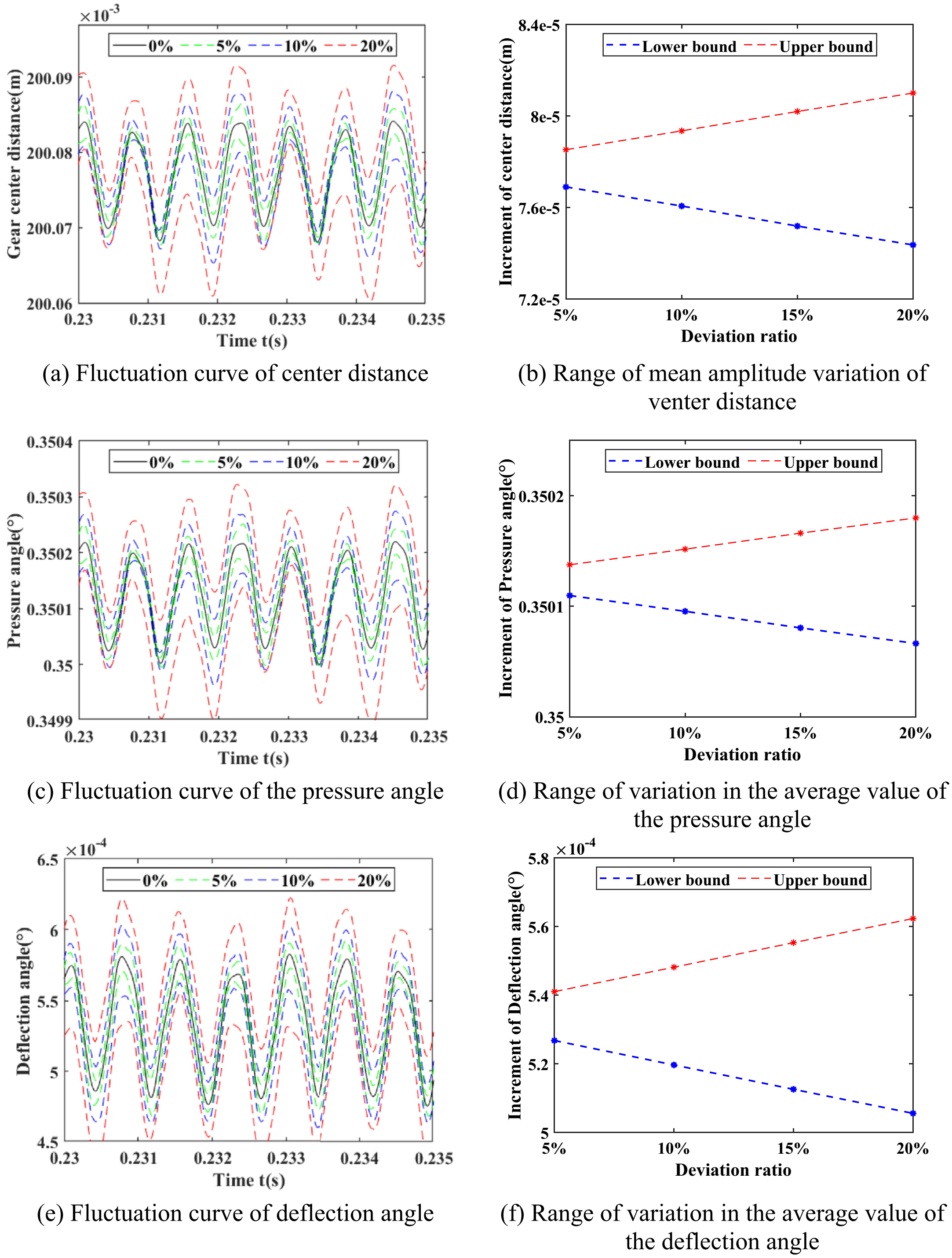
(a) The fluctuation curve of gear clearance

(b) The range of average variation in gear clearance

(c) Variation curve of bearing clearance

(d) Variation range of the average bearing clearance

Fig. 8. Variation curves and interval fluctuation ranges of gear and bearing clearances with uncertainty parameter δ_0.



(a) Fluctuation curve of center distance

(b) Range of mean amplitude variation of venter distance

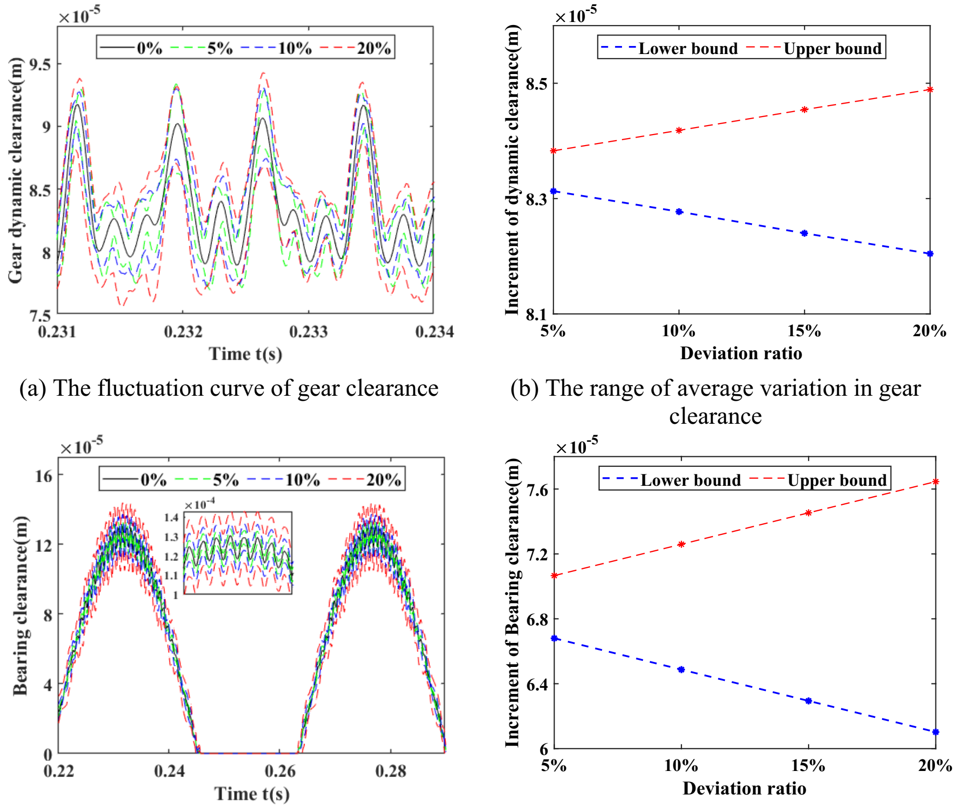
(c) Fluctuation curve of the pressure angle

(d) Range of variation in the average value of the pressure angle

(e) Fluctuation curve of deflection angle

(f) Range of variation in the average value of the deflection angle

Fig. 9. Variation of gear meshing parameters with δ_0 at 4000 rad/min for the driving gear.



(a) The fluctuation curve of gear clearance

(b) The range of average variation in gear clearance

(c) Variation curve of bearing clearance

(d) Variation range of the average bearing clearance

Fig. 10. Variation of gear clearance and bearing clearance with δ_0 at 1000 rad/min for the driving gear.

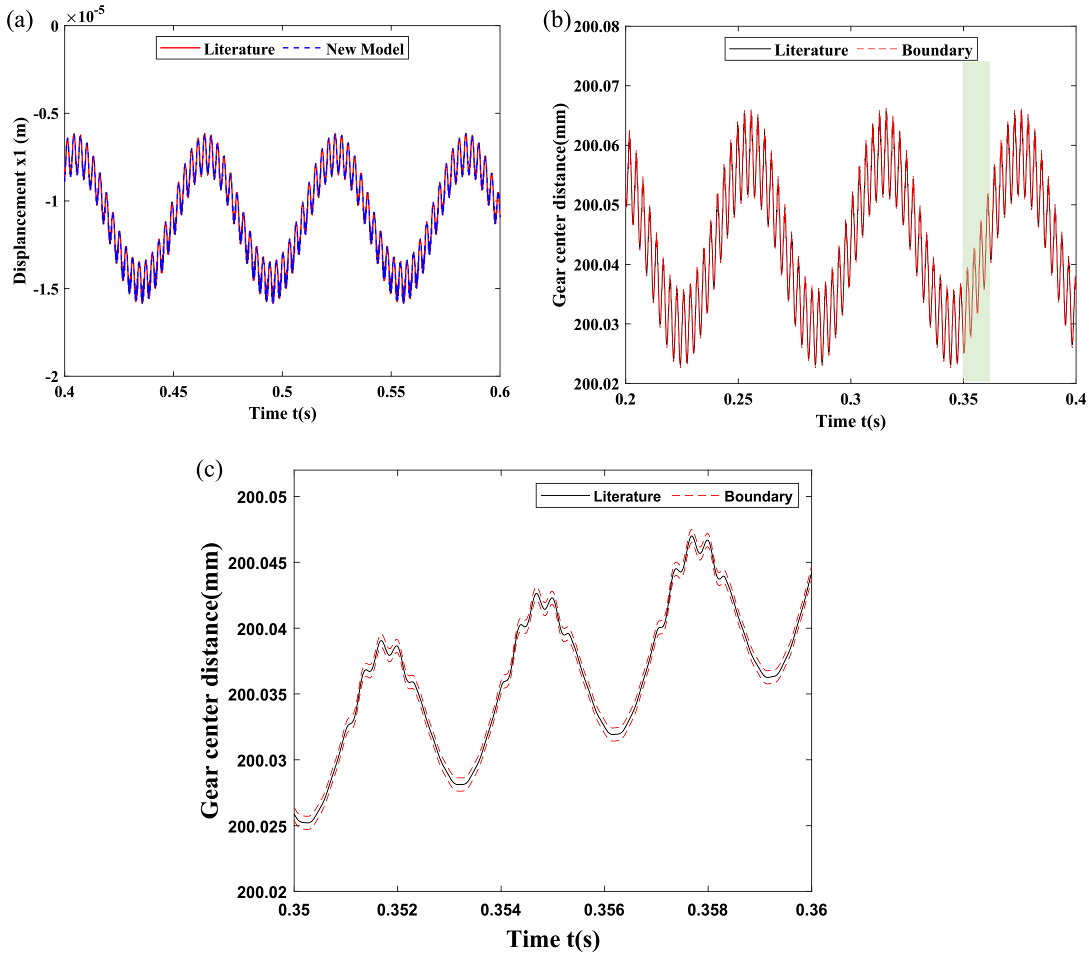


Fig. 11. Compares the theoretical results of the system with uncertain gear backlash to those reported in the literature.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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Author contributions

The authors' contributions are as follows: Jinzhou Song was in charge of the whole analyses and wrote the manuscript; Lei Hou assisted with sample analyses and revised the manuscript; Yifan Wang, Tao Zhang and Zhibin Zhang assisted with simulation analyses; Minghe Zhang, Yifan Jiang, Yi Chen and Rongzhou Lin participated in writing comments and editing. Yushu Chen revised the final manuscript. All authors read and approved the final manuscript.

Declarations

Competing interests

The authors declare no competing interests.

Additional information

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