



# OPEN Adaptive damage analysis method for ultimate bearing capacity of steel truss structures with box section

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Traditional damage analysis of steel truss structures requires detailed finite element meshing, and there is a coupling between stress-strain and damage solving, resulting in complex damage analysis and massive computations. In this paper, the element bearing ratio (EBR) reflecting the bearing state of components was defined, and the concept of the element damage factor was introduced based on the principle of deformation energy conservation. A homogeneous generalized yield function (HGYF) for box-section components was constructed to establish adaptive damage factors and the criterion under multi-force combinations. Combined with linear elastic iterative analysis, an adaptive damage evolution method for simulating steel truss structures with box section was proposed. Through numerical examples, the damage criterion in the method was able to adaptively determine whether component stiffness degradation occurred, eliminating the requirement to consider the interaction between stress-strain fields and damage, thereby reducing the difficulty of coupled calculations and enhancing computational efficiency in structural damage analysis. Based on the linear elastic iterative analysis, the calculated ultimate bearing capacity showed good agreement with experimental results.

**Keywords** Steel truss, Box-section, Adaptive damage, Ultimate bearing capacity, Damage factor, Homogeneous generalized yield function (HGYF)

Steel truss structures, characterized by their simplicity, high overall stiffness, and excellent compressive and torsional performance, are widely used in large-span and heavy-load bridge engineering. The analysis of their ultimate bearing capacity is fundamental for safety assessment and design, garnering significant academic attention. Box section is commonly used in steel truss structures, and this study mainly focuses on the adaptive damage analysis of box steel trusses.

Structural damage evolution analysis focuses on the initiation, propagation, and coalescence of material micro-defects, and the overall or local failure of structures, serving as a crucial method for determining the ultimate bearing capacity<sup>1,2</sup>. Current design and analysis of large-scale engineering structures predominantly employ the theory of continuum damage mechanics<sup>3–5</sup>, integrating material damage constitutive models to study the damage evolution process from microscopic material damage to macroscopic structural failure<sup>6–9</sup>. Various models for steel damage constitutive models have been proposed by scholars such as Lemaitre et al.<sup>10</sup>, Bonora et al.<sup>11</sup>, Celentano et al.<sup>12</sup>, Castiglioni et al.<sup>13</sup>, Vu et al.<sup>14</sup>, and Hosseini et al.<sup>15</sup>.

In the study of damage simulation in steel truss structures, Yang et al.<sup>16</sup> calibrated damage model parameters through steel plate experiments and developed a hybrid multiscale damage simulation model combining local microscopic and overall mesoscopic approaches. Li et al.<sup>17</sup> developed a finite element program for elastic damage analysis of steel trusses, establishing damage variables using Miner's linear damage accumulation theory and incorporating Lemaitre's<sup>10</sup> plastic damage evolution equation as the microscopic damage model. They characterized macroscopic damage through elastic modulus reduction, constructing a multiscale damage analysis model for steel trusses. Driemeier et al.<sup>18</sup> derived elastoplastic constitutive relations for damaged steel and an explicit tangent modulus, establishing a nonlinear analysis method for steel trusses capable of handling

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large strains, damage, and plasticity. Li et al.<sup>19</sup> established a prediction method for the structural damage state of steel frames under aftershocks by considering the fracture damage and failure of steel component welds. Gong et al.<sup>20</sup> conducted a study on the bearing capacity and damage mode identification of large-span truss structures in ultra-high voltage substations under earthquake action from a large-scale perspective using numerical methods. Zhao et al.<sup>21</sup> theoretically analyzed the mechanical properties and failure mechanisms of channel steel supports, and studied the fracture damage behavior of the structure under dynamic loads from a macroscopic perspective through experiments.

However, in the research on damage analysis of steel truss structures mentioned above, the damage evolution equation involves stress-strain relationships, and the damage field itself affects the outcomes of stress-strain. Therefore, precise structural damage evolution analysis must consider the coupling effects between stress-strain and damage. This theoretical complexity involves extensive nonlinear iterative analyses at the material level, leading to substantial computational demands. Moreover, stress-strain analysis requires finely divided finite element meshes and the results are highly sensitive to mesh refinement, which reduces computational efficiency and is challenging for the analysis of large-scale structures' ultimate bearing capacity.

To overcome the challenges posed by incorporating material damage constitutive models, one can consider establishing damage variables based on components or structures to simplify the complexity of material damage expressions. Li et al.<sup>22</sup>, Chen et al.<sup>23</sup>, Ma et al.<sup>24</sup>, and Diaz et al.<sup>25</sup> have proposed using indicators such as strength, displacement, and energy dissipation of components or structures to define damage variables, which to some extent address the complexity of material damage expressions. However, these studies primarily focus on the consequences of structural damage for assessment purposes, lacking descriptions of damage initiation mechanisms, evolution processes, and calculations of ultimate load-bearing capacity<sup>17</sup>. Thus, there is room for improvement in the computational efficiency and stability of existing steel truss structure damage evolution analysis methods.

Material damage typically leads to a reduction in elastic stiffness and a corresponding decrease in the elastic modulus during the damage phase. Therefore, the damage state can be described by reducing the elastic modulus. Based on this, this paper establishes damage factors and the criterion for steel components by using their elastic modulus as a variable and simulates the damage evolution process through adaptive stiffness degradation. This approach does not involve interactions between stress-strain fields and damage during analysis, thereby reducing the difficulties of coupled calculations and avoiding the nonlinear iterative process of calculating strain energy through stress-strain, which enhances the efficiency of structural damage analysis. Additionally, a method for calculating the load-bearing capacity of steel components and steel truss structures post-damage is proposed in this paper, and the ultimate load-bearing capacity of the steel truss structure is determined through linear elastic iterative analysis. The accuracy and correctness of the method are verified by comparing it with experimental results.

## Damage factor and damage criterion

### Traditional damage factor based on stress-strain

Kachanov<sup>26</sup> and Rabotnov et al.<sup>27</sup> defined the damage factor  $D$  from the perspective of material degradation:

$$D = \frac{A - \tilde{A}}{A} \quad (1)$$

Where:  $A$  is the cross-sectional area before damage, while  $\tilde{A}$  is the cross-sectional area after damage.  $D = 0$  indicates an undamaged state, and  $D = 1$  indicates a fully damaged state.

According to Eq. (1), the relationship between effective stress  $\tilde{\sigma}$  and nominal stress  $\sigma$  is given by:

$$\tilde{\sigma} = \sigma / (1 - D) \quad (2)$$

Lemaitre<sup>28</sup> proposed the strain equivalence hypothesis, stating that the equivalent relationship of strain  $\varepsilon$  under one-dimensional conditions is:

$$\varepsilon = \tilde{\sigma} / E = \sigma / \tilde{E} \quad (3)$$

Where:  $E$  is the initial elastic modulus of the material;  $\tilde{E}$  is the secant modulus of the damaged material.

Combining Eqs. (2) and (3):

$$\tilde{E} = (1 - D) E \quad (4)$$

Najar defined the damage factor  $D$  from the perspective of strain energy and its dissipative damage<sup>29</sup>.

$$\Delta w_\varepsilon = w_{\text{perf}} - w_\varepsilon, D = \frac{\Delta w_\varepsilon}{w_{\text{perf}}} = \frac{w_{\text{perf}} - w_\varepsilon}{w_{\text{perf}}} \quad (5)$$

Where:  $w_{\text{perf}}$  is the strain energy of the material in an undamaged state;  $w_\varepsilon$  and  $\Delta w_\varepsilon$  are the strain energy and dissipated energy after damage occurs, respectively.

In summary, the damage factor  $D$  is typically defined through strain energy. However, the calculation of strain energy relies on the resolution of stress-strain relations, which often involves extensive nonlinear iterative analyses, leading to prolonged computation times for structural damage assessments. To address this issue, the element bearing ratio (EBR) is introduced in this paper, and the damage factor is defined at the component level,

allowing for a comprehensive representation of the damage state and improving the computational efficiency of structural damage analyses.

### The EBR characterizing the bearing state of the component

From a material perspective, the bearing state at a specific point can be measured by the ratio of stress to material strength. Although this representation provides a detailed reflection of the bearing state at each force application point, it requires significant computational effort and time and may not comprehensively represent the overall bearing status of the component. To address this, the EBR<sup>30–35</sup> is introduced to reflect the bearing state of the component. The EBR is defined as the ratio of sectional load effects to resistance. For example, in a hinged truss structure subjected to concentrated loads at the nodes, the axial force is the governing internal force, and the EBR  $r_k^e$  can be defined as follows:

$$r_k^e = |N / N_u| \quad (6)$$

Where:  $N$  is the axial force acting on the cross-section,  $N_u$  is the compressive or tensile capacity of the cross-section, the subscript  $k$  is the iteration step, and the superscript  $e$  is the element number. When considering the combined effects of multiple internal forces, the generalized yield function (GYF)<sup>36,37</sup> can be used to calculate the EBR.

### Damage criterion and damage factors for components based on the EBR

Material damage typically results in a reduction of the elastic stiffness, with the elastic modulus decreasing during the damage stage. Thus, damage states can be described by the reduction in elastic modulus<sup>38</sup>. For isotropic steel, assuming isotropic characteristics of the damage, the macroscopic variation in mechanical properties (i.e., stiffness) can be used to define the damage. Furthermore, stiffness can be repeatedly measured during the damage process without causing additional harm to the material<sup>10</sup>. Therefore, as shown in Eq. (4), the damage factor can be expressed as the ratio of the elastic modulus before and after damage:

$$D = 1 - \tilde{E} / E \quad (7)$$

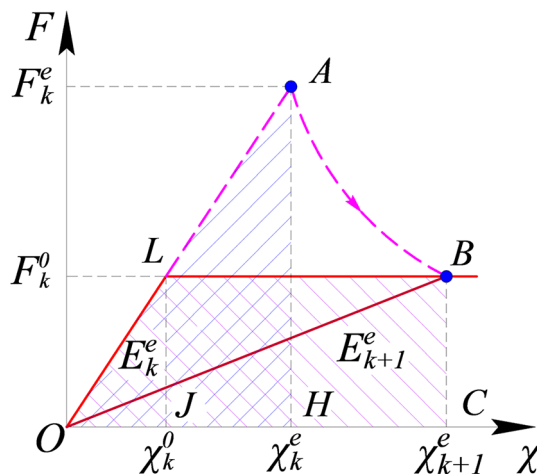
During the iterative calculation process, the elastic modulus of the component continuously changes. The post-damage elastic modulus  $\tilde{E}$  can be regarded as the elastic modulus  $E_{k+1}^e$  of the  $(k + 1)_{th}$  iteration. Thus, Eq. (7) can be expressed as:

$$D_k^e = 1 - E_{k+1}^e / E_0^e \quad (8)$$

Where:  $D_k^e$  is the damage factor of the  $k_{th}$  iteration element;  $E_0^e$  is the initial elastic modulus.

The elastic modulus of the damaged element is derived using the principle of strain energy conservation, which also facilitates the calculation of the element damage factor. This principle states that before any further damage, the total strain energy of the component is equal to the sum of the elastic strain energy after damage and the energy dissipated due to damage. As illustrated in Fig. 1, the vertical axis represents the generalized internal force on the characteristic cross-section of the component, while the horizontal axis denotes the corresponding deformation.

In Fig. 1, the area of triangle  $OAH$  represents the strain energy of the component before any further damage occurs, while the area of trapezoid  $LBCO$  denotes the sum of the elastic strain energy after damage and the energy dissipated due to damage (equivalent to the sum of triangle  $OLJ$  and rectangle  $LJCB$ ). The internal force at point  $A$  corresponds to  $F_k^e$ , with a deformation of  $\chi_k^e$ ; similarly, the internal force at points  $B$  and  $L$  corresponds



**Fig. 1.** Conservation of strain energy.

to  $F_k^0$ , with point  $L$  having a deformation of  $\chi_k^0$ , and point  $B$  having a deformation of  $\chi_{k+1}^e$ . Thus, according to the principle of strain energy conservation:

$$\frac{1}{2} F_k^e \chi_k^e = \frac{1}{2} F_k^0 \chi_k^0 + F_k^0 (\chi_{k+1}^e - \chi_k^0) \quad (9)$$

Thus, the deformation of  $\chi_{k+1}^e$  corresponding to point  $B$  after the damage is:

$$\chi_{k+1}^e = \frac{F_k^e \chi_k^e - F_k^0 \chi_k^0}{2F_k^0} + \chi_k^0 \quad (10)$$

Further combining Hooke Law with the relationship between internal forces and the element bearing ratio:

$$F_k^e = E_k^e \chi_k^e = r_k^e C_s, \quad F_k^0 = E_k^0 \chi_k^0 = r_k^0 C_s \quad (11)$$

Where:  $C_s$  represents the cross-sectional strength of the element. Taking a hinged truss structure as an example,  $F_k^e$  denotes the axial force on the cross-section, and  $C_s$  signifies the compressive or tensile strength of the section.  $r_k^0$  represents the reference bearing ratio (RBR), satisfying the following relationship:

$$r_k^0 = r_k^{\max} - d_k (r_k^{\max} - r_k^{\min}) \quad (12)$$

$$d_k = \frac{\bar{r}_k + r_k^{\min}}{\bar{r}_k + r_k^{\max}}, \quad \bar{r}_k = \frac{1}{N_e} \sum_{e=1}^{N_e} r_k^e \quad (13)$$

Where:  $r_k^{\min}$  and  $r_k^{\max}$  are the maximum and minimum EBRs of the elements at the  $k_{th}$  iteration step, respectively;  $d_k$  is the uniformity of the EBRs;  $\bar{r}$  is the mean bearing ratio;  $N_e$  is the total number of discrete elements in the structure.

The EBR  $r_k^0$  serves as a damage criterion for the components. Elements satisfying  $r_k^e > r_k^0$  will incur damage, and  $r_k^0$  can adaptively change based on the load states of the structural elements without human intervention, demonstrating adaptive characteristics.

Substitute Eq. (11) into Eq. (10):

$$\chi_{k+1}^e = \frac{C_s}{E_k^e} \frac{(r_k^e)^2 + (r_k^0)^2}{2r_k^0} \quad (14)$$

The element adjustment factor is defined as follows:

$$M_{0k}^e = \begin{cases} \frac{2(r_k^0)^2}{(r_k^e)^2 + (r_k^0)^2}, & r_k^e > r_k^0 \\ 1, & r_k^e \leq r_k^0 \end{cases} \quad (15)$$

Correspondingly, the element damage development factor is defined as:

$$D_{0k}^e = 1 - M_{0k}^e = \frac{(r_k^e)^2 - (r_k^0)^2}{(r_k^e)^2 + (r_k^0)^2} \quad (16)$$

The formula for calculating the element elastic modulus after stiffness degradation  $E_{k+1}^e$  is:

$$\begin{aligned} E_{k+1}^e &= \frac{2(r_k^0)^2}{(r_k^e)^2 + (r_k^0)^2} E_k^e = (1 - D_{0k}^e) E_k^e \\ &= (1 - D_{01}^e) (1 - D_{02}^e) \cdots (1 - D_{0k}^e) E_0^e = E_0^e \prod_{i=1}^k M_{0i}^e \end{aligned} \quad (17)$$

Where:  $D_{0k}^e$  is used to determine whether further damage occurs in the component. When  $D_{0k}^e = 0$ , and  $r_k^e \leq r_k^0$ , indicating no further damage; when  $0 < D_{0k}^e < 1$ , and  $r_k^e > r_k^0$ , indicating continued damage. Thus, the range of  $D_{0k}^e$  is  $[0, 1)$ , and:

$$D_{0k}^e = \frac{(r_k^e)^2 - (r_k^0)^2}{(r_k^e)^2 + (r_k^0)^2} \geq 0 \quad (18)$$

By substituting Eq. (17) into Eq. (8), the damage factor can be expressed as:

$$D_k^e = 1 - (1 - D_{01}^e)(1 - D_{02}^e) \cdots (1 - D_{0k}^e) = 1 - \prod_{i=1}^k M_{0k}^e \quad (19)$$

A larger  $D_{0k}^e$  indicates more evident damage to element  $e$ . Thus, the damage criterion and factor defined based on the EBR do not directly involve material strain, effectively reducing the computational complexity. This approach eliminates the requirement for fine finite element meshing of the cross-section, resulting in higher computational efficiency and stability compared to existing damage analysis methods for trusses.

### The EBR and damage factor of components

For box-shaped steel components subjected to multiple internal force combinations, Liu et al.<sup>39,40</sup> assessed the damage and failure of the components using the axial force-bending moment relations provided in GB50017<sup>41</sup>. However, the related equations from this code are non-homogeneous functions, and the internal force components do not maintain proportional variations with the loads, which does not satisfy the proportionality loading conditions required for plastic limit analysis. This may compromise the stability and accuracy of structural ultimate load capacity calculations<sup>31</sup>. To address this, an equivalent homogeneous generalized yield function (HGYF) was established based on the relevant equations for box-shaped steel components under multiple internal forces, enabling the definition of the EBR and facilitating the determination of the damage factor for box components.

### The correlation equation and HGYP of box-shaped steel components

The box section is a commonly used profile in steel trusses. The correlation equations for the in-plane stability ultimate bearing capacity of box-shaped steel components provided in GB50017<sup>41</sup> are as follows:

$$\frac{N}{\varphi A f_d} + \frac{\beta_m}{1 - 0.88N/N_{cr}} \frac{M}{\gamma W f_d} \leq 1 \quad (20)$$

Where:  $N$  and  $M$  represents the axial force and in-plane bending moment on the component, respectively;  $A$  is the cross-sectional area;  $\varphi$  is the overall stability coefficient for axially compressed members;  $f_d$  is the design value of the yield strength of the steel;  $f_y$  is the yield strength of the steel and  $f_y = 1.1f_d$ ;  $\gamma$  is the plastic development coefficient of the section;  $W$  is the section modulus;  $\beta_m$  is the equivalent moment coefficient; and  $N_{cr}$  is the buckling load for axially compressed members.

Based on the correlation equations presented in Eq. (20), the GYF for the box-shaped steel components under various internal force combinations can be expressed as:

$$f(n, m) = 1 \quad (21)$$

Where:  $f$  is the GYF.  $n$  and  $m$  are the dimensionless internal forces corresponding to axial force  $N$  and in-plane bending moment  $M$ , respectively, such that:

$$n = \frac{N}{\varphi A f_d}, m = \frac{\beta_m M}{\gamma W f_d}, a_E = \frac{\varphi A f_d}{N_{cr}} \quad (22)$$

From Eqs. (20) and (22), the GYF for the box-shaped steel components can be expressed as:

$$f(n, m) = n + \frac{m}{1 - 0.88a_E n} \quad (23)$$

### The HGYP of box-shaped steel components

Equation (23) indicates that the mechanical behavior of the box-shaped steel components under axial force and bending moment differs, leading to inconsistent power terms for the corresponding dimensionless internal forces. This implies that the GYF exhibits non-homogeneous characteristics, violating the proportional loading conditions required for plastic limit analysis. To address this, a corresponding HGYP is established through regression analysis.

#### The expression for the HGYP

By treating  $a_E$  as a variable for fitting, the expression for the HGYP equivalent to Eq. (23) is established, denoted as  $\bar{f}_{S_1}(n, m, a_E)$ :

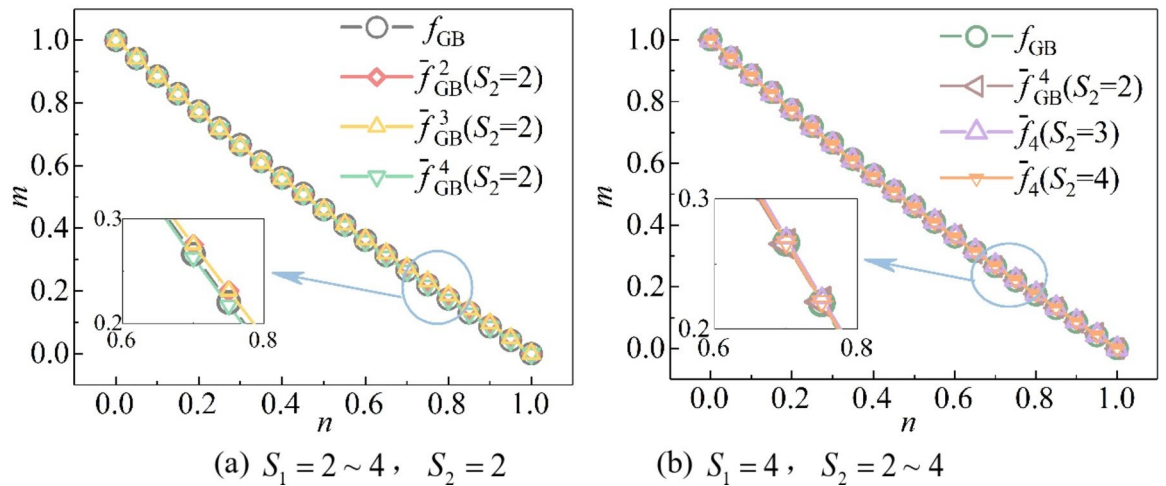
$$\bar{f}_{S_1}(n, m, a_E) = \sum_{i=1}^{S_1} c_q n^i m^{S_1-i} \quad (q = 1, 2, \dots, S_1 + 1) \quad (24)$$

Where:  $S_1$  represents the highest degree of the homogeneous polynomial  $\bar{f}_{S_1}$ ;  $c_q$  is a higher-order expression in terms of  $a_E$ . When  $i = 0$ , or  $i = S_1$ , then  $c_q = 1$ ; otherwise:

$$c_q = \sum_{j=0}^{S_2} k_{qj} a_E^j = k_{q0} a_E^0 + k_{q1} a_E^1 + \dots + k_{qS_2} a_E^{S_2} \quad (25)$$

| $S_2$ | $j$ | $S_1 = 2$ |         | $S_1 = 3$ |         | $S_1 = 4$ |         |  |
|-------|-----|-----------|---------|-----------|---------|-----------|---------|--|
|       |     | $q = 1$   | $q = 1$ | $q = 2$   | $q = 1$ | $q = 2$   | $q = 3$ |  |
| 2     | 0   | 2.01      | 2.96    | 3.10      | 4.09    | 5.62      | 4.34    |  |
|       | 1   | 1.50      | 3.27    | 1.22      | 2.52    | 11.62     | -1.09   |  |
|       | 2   | 0.78      | -1.42   | 5.67      | 1.73    | -3.49     | 13.11   |  |
| 3     | 0   | 2.00      | 3.01    | 2.99      | 3.98    | 6.09      | 3.94    |  |
|       | 1   | 1.60      | 2.50    | 2.97      | 4.02    | 4.64      | 5.22    |  |
|       | 2   | 0.51      | 0.68    | 0.75      | -2.13   | 15.13     | -4.55   |  |
|       | 3   | 0.19      | -1.47   | 3.50      | 2.58    | -12.81    | 12.55   |  |
| 4     | 0   | 2.00      | 3.00    | 3.00      | 4.00    | 5.99      | 4.01    |  |
|       | 1   | 1.61      | 2.64    | 2.61      | 3.40    | 7.70      | 3.07    |  |
|       | 2   | 0.46      | 0.00    | 2.58      | 0.90    | -0.36     | 6.52    |  |
|       | 3   | 0.28      | -0.34   | 0.43      | -2.32   | 12.84     | -6.02   |  |
|       | 4   | -0.04     | -0.58   | 1.61      | 2.50    | -13.31    | 9.72    |  |

**Table 1.** Undetermined parameters  $k_{qj}$  of HGYP with different  $S_1$  and  $S_2$



**Fig. 2.** Comparison of GYF and HGYP ( $a_E = 0.2$ ).

Where:  $S_2$  denotes the highest order in the expression of  $c_q$ ;  $k_{qj}$  represents the coefficients determined through regression analysis.

In regression analysis, the arrangement of data points significantly impacts the fitting accuracy of the HGYP. This study employs a comprehensive experimental method to arrange data points  $(n^i, m^i)$ ,  $i = 1, 2, 3 \dots N$  on the GYF defined in Eq. (23), with increased density near the extremum points, totaling 441 data points to enhance the fitting accuracy of the HGYP. The coefficients of the HGYP are determined using the least squares method<sup>31,32</sup>, as shown in Table 1. The HGYP expression can be derived from  $k_{qj}$ , Eqs. (24) and (25). Due to space constraints, only the HGYP for  $S_1 = 2 \sim 4$  and  $S_2 = 2$  is presented, denoted as  $\tilde{f}_2$ ,  $\tilde{f}_3$ ,  $\tilde{f}_4$ , as in Eqs. (26) to (28):

$$\tilde{f}_2 = m^2 + (2.01 + 1.50a_E + 0.78a_E^2)mn + n^2 \quad (26)$$

$$\tilde{f}_3 = m^3 + (2.96 + 3.27a_E - 1.42a_E^2)m^2n + (3.10 + 1.22a_E + 5.67a_E^2)mn^2 + n^3 \quad (27)$$

$$\begin{aligned} \tilde{f}_4 = m^4 + (4.09 + 2.52a_E + 1.73a_E^2)m^3n + (5.62 + 11.62a_E - 3.49a_E^2)m^2n^2 \\ + (4.34 - 1.09a_E + 13.11a_E^2)mn^3 + n^4 \end{aligned} \quad (28)$$

#### Fitting error of the HGYP

This section compares the HGYP curve with the GYF curve at  $a_E = 0.2$ , as shown in Fig. 2. In Fig. 2a,  $\tilde{f}_{S_1}$  represents the second to fourth-order HGYP ( $S_2 = 2$ ),  $f_{GB}$  denotes the GYF from GB50017, and  $\tilde{f}_{GB}^2(*) \sim \tilde{f}_{GB}^4(*)$  represent the HGYPs with maximum orders  $S_1$  of 2, 3, and 4, respectively. For the same maximum order  $S_2$ , the different-order HGYPs closely overlap with the original GYF. Specifically, when the maximum order of  $c_q$  is 2, the fitting accuracy of the different-order HGYPs is consistent and aligns well with the GYF. Figure 2b



presents the comparison curves of the fourth-order HGYF with the GYF for  $S_2 = 2 \sim 4$ . It shows that under the same  $S_1$ , the yield curves for  $S_2 = 2 \sim 4$  nearly coincide, indicating that  $S_2$  has a minimal impact on the fitting accuracy of the HGYF. The fitting errors between the GYF and the corresponding HGYFs for different  $S_1$  and  $S_2$  are summarized in Table 2.

Table 2 indicates that the root mean square errors (RMSE) of different-order HGYFs with varying maximum orders  $S_2$  do not exceed 3%, demonstrating high fitting accuracy. When the order  $S_1$  of the HGYF remains constant, changes in the maximum order  $S_2$  of  $c_q$  have minimal impact on fitting accuracy. Conversely, for a fixed maximum order  $S_2$ , a higher  $S_1$  results in improved fitting accuracy. Therefore, considering both the complexity of the expression and the fitting precision, this study selects the HGYF with  $S_1 = 2$  and  $S_2 = 2$  for structural analysis.

### The EBR and damage factor of box-shaped steel components

The EBR of box-shaped steel components under axial force and moment interaction is defined using the 2nd-order HGYF, as shown in Eq. (26).

$$r_k^e = \sqrt[2]{f_2} \quad (29)$$

Different ends of the same element often have varying values of  $r_k^e$ , requiring the maximum value to be used. Substitute  $r_k^e$  into Eq. (19), and yield the damage factor of the box-shaped steel components.

$$D_k^e = 1 - \prod_{i=1}^k g(r_i^e, r_i^0), \quad g(r_i^e, r_i^0) = \frac{2(r_i^0)^2}{(r_i^e)^2 + (r_i^0)^2} \leq 1 \quad (30)$$

Based on the determined internal forces, the EBR  $r_k^e$  can be calculated using Eq. (29). Subsequently, the damage criterion  $r_k^0$  is computed using Eq. (12). Components with  $r_k^e > r_k^0$  will experience damage, and the damage factor is assessed using Eq. (30). Finally, Eq. (8) is used to reduce the elastic modulus of the component, allowing simulation of structural damage evolution through linear elastic iterative analysis.

### Linear elastic iterative analysis of the structure

The 2nd order HGYF in Eq. (26) is used to define  $r_k^e$  for the each members. At each iterative step  $k$ , the linear elastic finite element analysis is used to obtain the stability ultimate capacity of the structure, where the maximum element bearing ratio of all members,  $r_k^{\max}$ , can measure the bearing and damage states of the structure. The stability ultimate capacity of the iterative step can be determined as:

$$P_k^L = P_0 / r_k^{\max} \quad (31)$$

Repeat the above iterative process until the stability ultimate capacity of the two adjacent steps meets the convergence criterion:

$$\left| \frac{P_k^L - P_{k-1}^L}{P_{k-1}^L} \right| \leq \varepsilon, \quad k \geq 2 \quad (32)$$

Where  $\varepsilon$  is the convergence tolerance, generally taken as 0.001 ~ 0.01.

### Analysis and verification

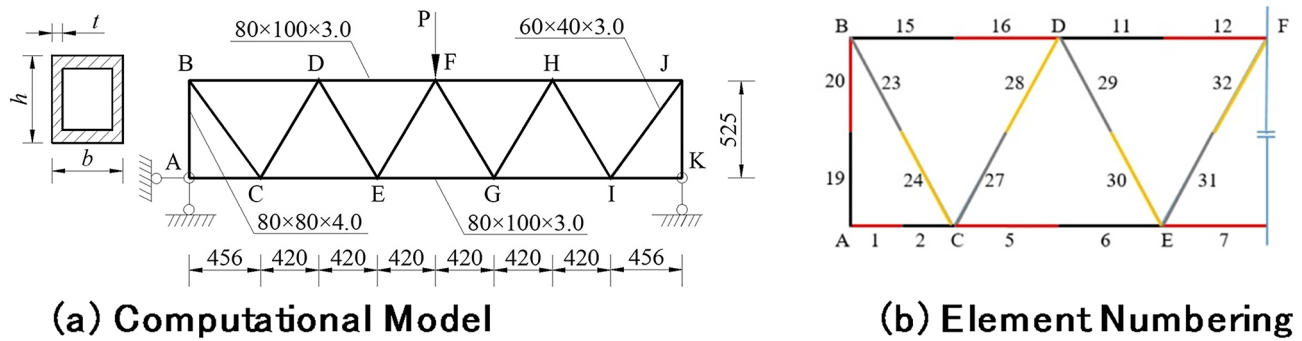
Based on box-section HGYF, the damage evolution of the steel truss structure in reference<sup>39</sup> and [40] is analyzed using the method presented in this paper. The ultimate bearing capacity of the structure is also solved through the linear elastic iterative method<sup>30–35</sup> and compared with experimental results to demonstrate the correctness and accuracy of the method. A finite element model is established using the ANSYS software, with beam189 elements selected for discretization. Rigid connections are applied at all component intersection points. The PC configuration used includes a CPU@2.50 GHz and 8GB of memory.

### Warren steel truss structure (triangular web system)

As shown in Fig. 3, the steel truss structure consisting of 19 members in reference<sup>40</sup> has a yield strength of 300 MPa and an elastic modulus of 206 GPa. Referring to the full-size test model<sup>40</sup>, a finite element model is established through the centerline of the member, as shown in Fig. 3a. The length of the left and right side spans is 456 mm, the length of the middle span is 420 mm, and the height of the truss is 525 mm. The structure is

| $S_2 = 2$ | $S_2 = 3$ | $S_2 = 4$ |
|-----------|-----------|-----------|
| 0.028     | 0.028     | 0.028     |
| 0.018     | 0.018     | 0.018     |
| 0.013     | 0.011     | 0.011     |

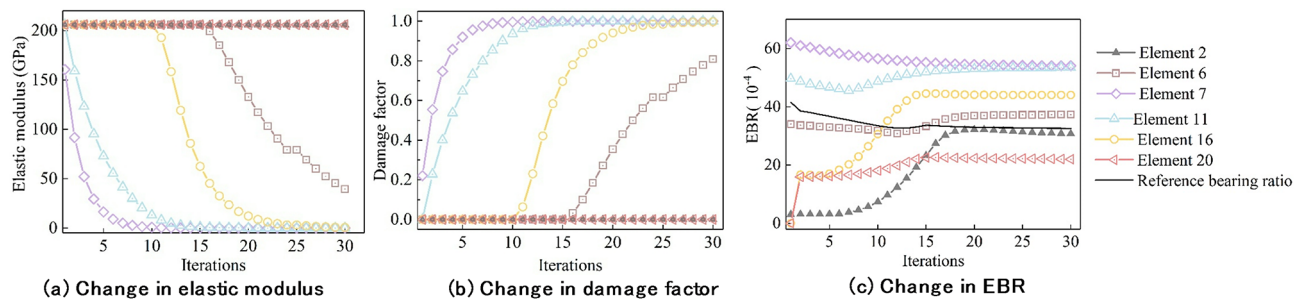
**Table 2.** Residual mean square error of HGYFs.



**Fig. 3.** Idealization model of Warren Steel Truss.

| Member category          | Member Number          | $l$ (mm) | $b \times h \times t$ (mm) |
|--------------------------|------------------------|----------|----------------------------|
| Top and bottom chords    | DF, FH, CE, EG, GI     | 840      | $80 \times 100 \times 3.0$ |
|                          | BD, HJ                 | 876      | $80 \times 100 \times 3.0$ |
|                          | AC, IK                 | 456      | $80 \times 100 \times 3.0$ |
| Support vertical members | AB, KJ                 | 525      | $80 \times 80 \times 4.0$  |
| Web members              | BC, IJ                 | 659.39   | $60 \times 40 \times 3.0$  |
|                          | CD, DE, EF, FG, GH, HI | 672.33   | $60 \times 40 \times 3.0$  |

**Table 3.** Section sizes<sup>40</sup>.



**Fig. 4.** Damage evolution of elements.

subjected to a vertical concentrated load at mid-span. All members use a box section, with geometric dimensions provided in Fig. 3a and Table 3.

#### Stiffness damage and evolution of elements

The method presented in this paper is used to analyze the damage evolution of the steel truss structure shown in Fig. 3a. Each member is divided into 2 elements, as illustrated in Fig. 3b. Due to symmetry, only the left half-span structure is analyzed. Iterative changes in the elastic modulus, damage factors, and EBRs for some elements are shown in Fig. 4a–c.

According to the relationship between the bearing ratio of each element and the reference bearing ratio, elements can be divided into three categories: low-bearing units, Class 1 high-stressed elements, and Class 2 high-stressed elements. The EBR of low-bearing elements is always lower than the reference bearing ratio, the EBR of Class 1 high-stressed elements is always higher than the reference bearing ratio, and the EBR of Class 2 high-stressed elements is initially lower than the reference bearing ratio, and then gradually higher than the reference bearing ratio.

For low-stressed Element 20, the EBR remains below the reference bearing ratio throughout the iteration process, as shown in Table 4. According to Eqs. (15) to (19), the damage parameters for each iteration step are as follows:

$$M_{0k}^e = 1, D_{0k}^e = 0, \prod_{i=1}^k M_{0k}^e = 1, D_k^e = 0 \quad (33)$$



| Steps J | $r_k^0(10^{-4})$ | $r_k^e(10^{-4})$ | $M_{0k}^e$ | $D_{0k}^e$ | $\prod_{i=1}^k M_{0k}^e$ | $D_k^e$ |
|---------|------------------|------------------|------------|------------|--------------------------|---------|
| 2       | 49.56            | 0                | 1          | 0          | 1                        | 0       |
| 3       | 38.57            | 15.88            | 1          | 0          | 1                        | 0       |
| 4       | 38.03            | 15.95            | 1          | 0          | 1                        | 0       |
| 5       | 37.39            | 16.04            | 1          | 0          | 1                        | 0       |
| 6       | 36.72            | 16.16            | 1          | 0          | 1                        | 0       |
| 7       | 36.08            | 16.35            | 1          | 0          | 1                        | 0       |
| 8       | 35.42            | 16.61            | 1          | 0          | 1                        | 0       |
| 9       | 34.76            | 16.96            | 1          | 0          | 1                        | 0       |
| 10      | 34.12            | 17.45            | 1          | 0          | 1                        | 0       |
| 11      | 33.52            | 18.09            | 1          | 0          | 1                        | 0       |
| 12      | 33.02            | 18.86            | 1          | 0          | 1                        | 0       |
| 13      | 32.72            | 19.84            | 1          | 0          | 1                        | 0       |
| 14      | 32.67            | 21.04            | 1          | 0          | 1                        | 0       |
| 15      | 33.05            | 22.09            | 1          | 0          | 1                        | 0       |
| 16      | 33.72            | 22.57            | 1          | 0          | 1                        | 0       |
| 17      | 33.53            | 22.66            | 1          | 0          | 1                        | 0       |
| 18      | 33.33            | 22.54            | 1          | 0          | 1                        | 0       |
| 19      | 33.18            | 22.44            | 1          | 0          | 1                        | 0       |
| 20      | 33.07            | 22.38            | 1          | 0          | 1                        | 0       |
| 21      | 32.99            | 22.34            | 1          | 0          | 1                        | 0       |
| 22      | 32.93            | 22.31            | 1          | 0          | 1                        | 0       |
| 23      | 32.88            | 22.29            | 1          | 0          | 1                        | 0       |
| 24      | 32.84            | 22.26            | 1          | 0          | 1                        | 0       |
| 25      | 32.80            | 22.23            | 1          | 0          | 1                        | 0       |
| 26      | 32.76            | 22.19            | 1          | 0          | 1                        | 0       |
| 27      | 32.72            | 22.15            | 1          | 0          | 1                        | 0       |
| 28      | 32.69            | 22.11            | 1          | 0          | 1                        | 0       |
| 29      | 32.65            | 22.07            | 1          | 0          | 1                        | 0       |
| 30      | 32.62            | 22.03            | 1          | 0          | 1                        | 0       |

**Table 4.** Damage parameters for element 20 (low-stressed element). Low-stressed elements are those where the EBR remains below the reference value throughout the entire iteration process.

For Class 1 high-stressed Element 7, the EBR remains above the reference value throughout the iteration process, as shown in Table 5. Similarly, the damage parameters for iteration steps 2 and 3 are as follows:

$$M_{02}^e = \frac{2(r_2^0)^2}{(r_2^e)^2 + (r_2^0)^2} = \frac{2 \times (49.56 \times 10^{-4})^2}{(62.01 \times 10^{-4})^2 + (49.56 \times 10^{-4})^2} = 0.779, \quad (34)$$

$$D_{02}^e = 1 - M_{02}^e = 0.221, \quad \prod_{i=2}^2 M_{0k}^e = 0.779, \quad D_2^e = 1 - \prod_{i=2}^2 M_{0k}^e = 0.221$$

$$M_{03}^e = \frac{2(r_3^0)^2}{(r_3^e)^2 + (r_3^0)^2} = \frac{2 \times (38.57 \times 10^{-4})^2}{(61.00 \times 10^{-4})^2 + (38.57 \times 10^{-4})^2} = 0.571, \quad (35)$$

$$D_{02}^e = 1 - M_{02}^e = 0.429, \quad \prod_{i=2}^3 M_{0k}^e = 0.779 \times 0.571 = 0.445, \quad D_3^e = 0.555$$

The damage parameters for other iteration steps of Element 7 are calculated similarly. For Class 2 high-stressed Element 16, the EBR initially falls below the reference value, with damage parameters calculated as for Element 20. As internal force redistribution occurs, it later exceeds the reference value, at which point the damage parameters are calculated as for Element 7. The variation trends of damage parameters for all three types of elements are detailed in Tables 4, 5 and 6.

From Fig. 4, Tables 4, 5 and 6, it can be observed that Element 2 and Element 20 maintain EBRs below the reference throughout the iteration, experiencing no damage with a damage factor consistently at 0 and unaltered elastic modulus. Element 7 and Element 11 have EBRs above the reference, consistently undergoing damage where the damage factor increases and elastic modulus decreases, reaching a plastic limit state with a damage

| Steps J | $r_k^0(10^{-4})$ | $r_k^e(10^{-4})$ | $M_{Ok}^e$ | $D_{Ok}^e$ | $\prod_{i=1}^k M_{Ok}^e$ | $D_k^e$ |
|---------|------------------|------------------|------------|------------|--------------------------|---------|
| 2       | 49.56            | 62.01            | 0.779      | 0.221      | 0.779                    | 0.221   |
| 3       | 38.57            | 61.00            | 0.571      | 0.429      | 0.445                    | 0.555   |
| 4       | 38.03            | 60.39            | 0.568      | 0.432      | 0.253                    | 0.747   |
| 5       | 37.39            | 59.67            | 0.564      | 0.436      | 0.143                    | 0.857   |
| 6       | 36.72            | 58.97            | 0.559      | 0.441      | 0.080                    | 0.920   |
| 7       | 36.08            | 58.34            | 0.553      | 0.447      | 0.044                    | 0.956   |
| 8       | 35.42            | 57.79            | 0.546      | 0.454      | 0.024                    | 0.976   |
| 9       | 34.76            | 57.31            | 0.538      | 0.462      | 0.013                    | 0.987   |
| 10      | 34.12            | 56.90            | 0.529      | 0.471      | 0.007                    | 0.993   |
| 11      | 33.52            | 56.54            | 0.520      | 0.480      | 0.004                    | 0.996   |
| 12      | 33.02            | 56.22            | 0.513      | 0.487      | 0.002                    | 0.998   |
| 13      | 32.72            | 55.92            | 0.510      | 0.490      | 0.001                    | 0.999   |
| 14      | 32.67            | 55.65            | 0.513      | 0.487      | 0                        | 1       |
| 15      | 33.05            | 55.41            | 0.525      | 0.475      | 0                        | 1       |
| 16      | 33.72            | 55.20            | 0.543      | 0.457      | 0                        | 1       |
| 17      | 33.53            | 55.01            | 0.542      | 0.458      | 0                        | 1       |
| 18      | 33.33            | 54.85            | 0.539      | 0.461      | 0                        | 1       |
| 19      | 33.18            | 54.72            | 0.538      | 0.462      | 0                        | 1       |
| 20      | 33.07            | 54.61            | 0.537      | 0.463      | 0                        | 1       |
| 21      | 32.99            | 54.51            | 0.536      | 0.464      | 0                        | 1       |
| 22      | 32.93            | 54.44            | 0.536      | 0.464      | 0                        | 1       |
| 23      | 32.88            | 54.37            | 0.536      | 0.464      | 0                        | 1       |
| 24      | 32.84            | 54.31            | 0.535      | 0.465      | 0                        | 1       |
| 25      | 32.80            | 54.27            | 0.535      | 0.465      | 0                        | 1       |
| 26      | 32.76            | 54.22            | 0.535      | 0.465      | 0                        | 1       |
| 27      | 32.72            | 54.18            | 0.534      | 0.466      | 0                        | 1       |
| 28      | 32.69            | 54.15            | 0.534      | 0.466      | 0                        | 1       |
| 29      | 32.65            | 54.12            | 0.534      | 0.466      | 0                        | 1       |
| 30      | 32.62            | 54.09            | 0.533      | 0.467      | 0                        | 1       |

**Table 5.** Damage parameters for element 7 (Class 1 of high-stressed element).

factor near 1.0, indicating significant stiffness degradation. Element 6 and Element 16 initially have EBRs below the reference with no damage, but as internal force redistributes, their ratio exceeds the reference, and damage begins after steps 10 and 15, respectively, with a reduction in elastic modulus.

At the final iteration step, reaching the plastic limit state, Element 2 and Element 20, distant from concentrated loads, remain undamaged with 0 stiffness degradation, depicted as low-stress blue areas in Fig. 5. Upper chord Element 11 near concentrated loads suffer the most damage with 100% stiffness degradation, appearing as yellow-orange-red on the map.

Compared with experimental results, when the steel truss reaches its ultimate load, noticeable plastic deformation occurs in upper chord Element 16, Element 11, and Element 12, as shown in Fig. 5. This indicates a good agreement between the damage evolution analysis and experimental outcomes.

*Structural ultimate bearing capacity analysis*

The impact of finite element mesh discretization on ultimate bearing capacity results is first analyzed, as shown in Table 7, where NE represents the number of elements per member. It is evident that when each member is divided into two elements, the calculation of ultimate load capacity achieves stable convergence. Further increasing the mesh density has minimal impact on the results, indicating that the method used in this paper can achieve high accuracy without the need for a fine mesh and has low dependency on the mesh discretization scheme.

**Modified Warren truss structure (warren with vertical members)**

As shown in Fig. 6, the modified Warren steel truss structure in reference<sup>39</sup>, subjected to a vertical concentrated load at mid-span, has upper and lower chords, diagonal members, and vertical members with yield strengths of 297 MPa, 298 MPa, and 302 MPa, respectively, and an elastic modulus of 206 GPa. Referring to the full-size test model<sup>39</sup>, a finite element model is established through the centerline of the member, as shown in Fig. 6 (a). The span length is 800 mm and the height of the truss is 1000 mm. All members have box-shaped cross-sections, with geometric dimensions detailed in Fig. 7; Table 8.

| Steps J | $r_k^0(10^{-4})$ | $r_k^e(10^{-4})$ | $M_{0k}^e$ | $D_{0k}^e$ | $\prod_{i=1}^k M_{0k}^e$ | $D_k^e$ |
|---------|------------------|------------------|------------|------------|--------------------------|---------|
| 2       | 49.56            | 0                | 1          | 0          | 1                        | 0       |
| 3       | 38.57            | 16.65            | 1          | 0          | 1                        | 0       |
| 4       | 38.03            | 16.62            | 1          | 0          | 1                        | 0       |
| 5       | 37.39            | 16.42            | 1          | 0          | 1                        | 0       |
| 6       | 36.72            | 17.06            | 1          | 0          | 1                        | 0       |
| 7       | 36.08            | 18.33            | 1          | 0          | 1                        | 0       |
| 8       | 35.42            | 20.22            | 1          | 0          | 1                        | 0       |
| 9       | 34.76            | 22.91            | 1          | 0          | 1                        | 0       |
| 10      | 34.12            | 26.43            | 1          | 0          | 1                        | 0       |
| 11      | 33.52            | 30.60            | 1          | 0          | 1                        | 0       |
| 12      | 33.02            | 35.18            | 0.937      | 0.063      | 0.937                    | 0.063   |
| 13      | 32.72            | 39.20            | 0.821      | 0.179      | 0.769                    | 0.231   |
| 14      | 32.67            | 42.09            | 0.752      | 0.248      | 0.578                    | 0.422   |
| 15      | 33.05            | 44.05            | 0.720      | 0.280      | 0.417                    | 0.583   |
| 16      | 33.72            | 44.52            | 0.729      | 0.271      | 0.304                    | 0.696   |
| 17      | 33.53            | 44.52            | 0.724      | 0.276      | 0.220                    | 0.780   |
| 18      | 33.33            | 44.45            | 0.720      | 0.280      | 0.158                    | 0.842   |
| 19      | 33.18            | 44.31            | 0.718      | 0.282      | 0.114                    | 0.886   |
| 20      | 33.07            | 44.19            | 0.718      | 0.282      | 0.082                    | 0.918   |
| 21      | 32.99            | 44.10            | 0.718      | 0.282      | 0.059                    | 0.941   |
| 22      | 32.93            | 44.04            | 0.717      | 0.283      | 0.042                    | 0.958   |
| 23      | 32.88            | 44.02            | 0.716      | 0.284      | 0.030                    | 0.970   |
| 24      | 32.84            | 44.01            | 0.715      | 0.285      | 0.022                    | 0.978   |
| 25      | 32.80            | 44.02            | 0.714      | 0.286      | 0.015                    | 0.985   |
| 26      | 32.76            | 44.04            | 0.712      | 0.288      | 0.011                    | 0.989   |
| 27      | 32.72            | 44.05            | 0.711      | 0.289      | 0.008                    | 0.992   |
| 28      | 32.69            | 44.06            | 0.710      | 0.290      | 0.006                    | 0.994   |
| 29      | 32.65            | 44.06            | 0.709      | 0.291      | 0.004                    | 0.996   |
| 30      | 32.62            | 44.06            | 0.708      | 0.292      | 0.003                    | 0.997   |

Table 6. Damage parameters for element 16 (Class 2 of high-stressed element).

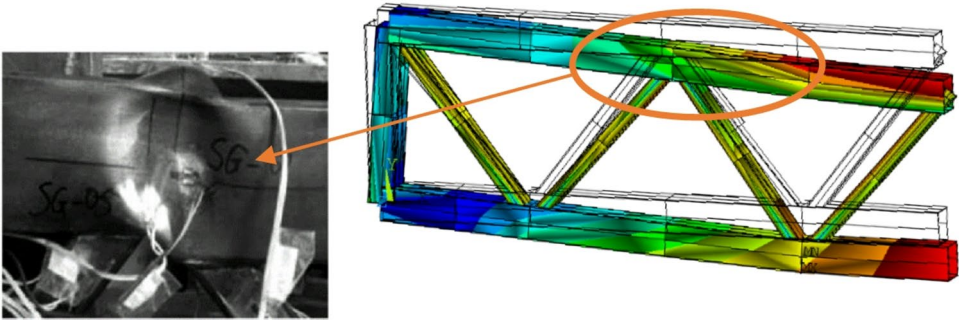


Fig. 5. Schematic diagram of damage location<sup>40</sup>.

| Experimental results <sup>40</sup> | NE = 2 | NE = 3 | NE = 4 | NE = 5 |
|------------------------------------|--------|--------|--------|--------|
| 180                                | 184.97 | 186.16 | 185.34 | 185.08 |

Table 7. Ultimate bearing capacity/kN.

| Member category       | Member number                                  | l (mm)  | b × h × t (mm)  | b / t |
|-----------------------|------------------------------------------------|---------|-----------------|-------|
| Top and bottom chords | BC, CF, FG, GJ, JK, KN, AD, DE, EH, HI, IL, LM | 800     | 100 × 100 × 4.0 | 25    |
| End vertical member   | AB, MN                                         | 1000    | 60 × 40 × 3.0   | 20    |
| Vertical web member   | CD, EF, GH, IJ, KL                             | 1000    | 60 × 40 × 3.0   | 20    |
| Diagonal web member   | AC, CE, EG, GI, IK, KM                         | 1280.62 | 80 × 80 × 4.0   | 20    |

Table 8. Section sizes of revised Warren steel truss [39].

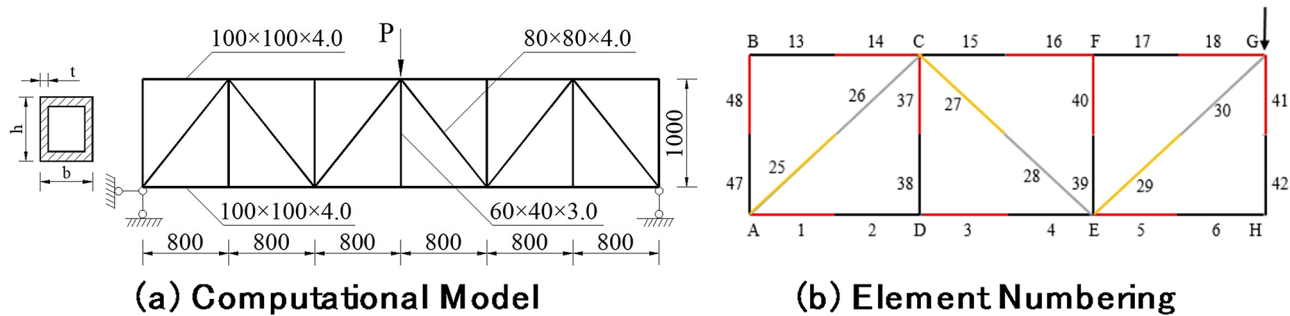


Fig. 6. Idealization model of revised Warren steel truss.

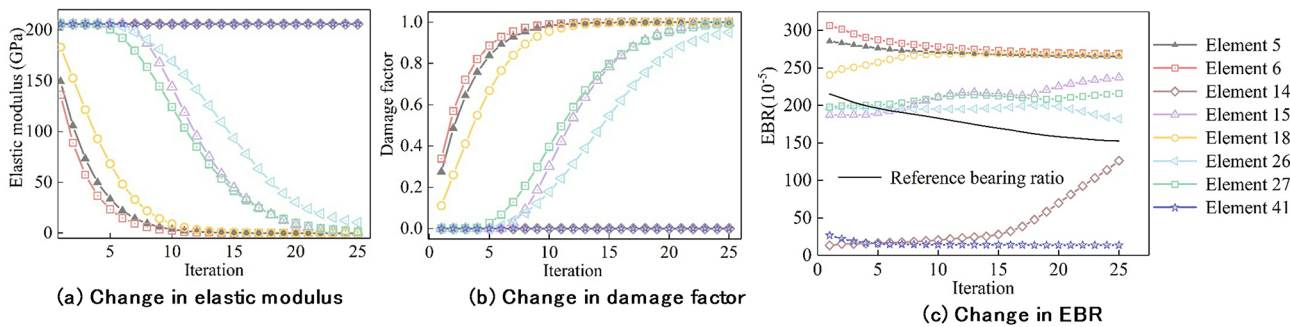


Fig. 7. Damage evolution of elements.

Stiffness damage and evolution of elements

Utilizing the methodology delineated in this paper, the damage evolution of the modified Warren steel truss structure depicted in Fig. 6a is investigated. Each truss member is subdivided into 2 elements, as illustrated in Fig. 6b. Analyzing only the left half-span due to symmetry, the iterative variations of elastic modulus, damage factor, and EBRs for select elements are presented in Fig. 7a–c. Damage parameters for Element 14, Element 5, and Element 15 are computed concerning Example 1 in Section “Warren steel truss structure (triangular web system)”, with detailed outcomes provided in Tables 9, 10 and 11. It is evident that elements 14 and 41 maintain EBRs below the benchmark throughout iterations, incurring no damage, with a damage factor of zero, and an unaltered elastic modulus. Conversely, Element 5, Element 6, and Element 18 consistently exhibit EBRs surpassing the benchmark, leading to progressive damage, increasing damage factors, and a decrement in elastic modulus. By the final iteration, as the structure approaches the plastic limit state, the damage factor nears 1.0, indicating pronounced stiffness degradation. Initially, Element 15, Element 26, and Element 27 have EBRs beneath the benchmark, thus avoiding damage. However, subsequent internal force redistribution results in EBRs exceeding the benchmark, initiating damage post the fourth step with a consequent reduction in elastic modulus. Thus, at the termination point of the plastic limit state iteration, diverse members manifest varying damage extents, with not all members concurrently reaching failure.

Comparing experimental results, when the test steel truss reaches its ultimate load, significant plastic deformation is observed, particularly in the severely damaged bottom chord Element 5 and Element 6, top chord Element 15 and Element 18, and diagonal web Element 26 and Element 27. This results in noticeable outward bulging, as illustrated for element 5 in Fig. 8. A similar deformation pattern is observed near node C in Element 15, indicating a strong correlation between the damage evolution analysis results presented in this study and the experimental outcomes.

| Steps J | $r_k^0(10^{-4})$ | $r_k^e(10^{-4})$ | $M_{0k}^e$ | $D_{0k}^e$ | $\prod_{i=1}^k M_{0k}^e$ | $D_k^e$ |
|---------|------------------|------------------|------------|------------|--------------------------|---------|
| 2       | 215              | 13               | 1          | 0          | 1                        | 0       |
| 3       | 210              | 15               | 1          | 0          | 1                        | 0       |
| 4       | 204              | 15               | 1          | 0          | 1                        | 0       |
| 5       | 200              | 16               | 1          | 0          | 1                        | 0       |
| 6       | 196              | 16               | 1          | 0          | 1                        | 0       |
| 7       | 193              | 17               | 1          | 0          | 1                        | 0       |
| 8       | 191              | 18               | 1          | 0          | 1                        | 0       |
| 9       | 188              | 18               | 1          | 0          | 1                        | 0       |
| 10      | 186              | 19               | 1          | 0          | 1                        | 0       |
| 11      | 183              | 21               | 1          | 0          | 1                        | 0       |
| 12      | 180              | 22               | 1          | 0          | 1                        | 0       |
| 13      | 178              | 23               | 1          | 0          | 1                        | 0       |
| 14      | 175              | 25               | 1          | 0          | 1                        | 0       |
| 15      | 172              | 25               | 1          | 0          | 1                        | 0       |
| 16      | 170              | 28               | 1          | 0          | 1                        | 0       |
| 17      | 167              | 32               | 1          | 0          | 1                        | 0       |
| 18      | 165              | 39               | 1          | 0          | 1                        | 0       |
| 19      | 162              | 48               | 1          | 0          | 1                        | 0       |
| 20      | 160              | 58               | 1          | 0          | 1                        | 0       |
| 21      | 158              | 70               | 1          | 0          | 1                        | 0       |
| 22      | 157              | 81               | 1          | 0          | 1                        | 0       |
| 23      | 156              | 93               | 1          | 0          | 1                        | 0       |
| 24      | 154              | 103              | 1          | 0          | 1                        | 0       |
| 25      | 153              | 114              | 1          | 0          | 1                        | 0       |
| 26      | 153              | 126              | 1          | 0          | 1                        | 0       |
| 27      | 32.72            | 22.15            | 1          | 0          | 1                        | 0       |
| 28      | 32.69            | 22.11            | 1          | 0          | 1                        | 0       |
| 29      | 32.65            | 22.07            | 1          | 0          | 1                        | 0       |
| 30      | 32.62            | 22.03            | 1          | 0          | 1                        | 0       |

**Table 9.** Damage parameters for element 14 (low load-stressed element).

*Structural ultimate bearing capacity analysis*

The influence of finite element mesh discretization on the ultimate load capacity calculations is initially examined, with findings detailed in Table 12, where NE represents the number of elements per structural member. The results demonstrate that dividing each member into two elements yields stable convergence in calculating the ultimate load capacity. Further refinement of mesh density has negligible impact on the results, suggesting that the proposed method can achieve high accuracy without the necessity for fine mesh discretization, and exhibits minimal dependency on the discretization scheme.

It is evident that the proposed method accurately identifies member damage by establishing damage factors and the criterion. It stimulates the damage evolution process through stiffness degradation, avoiding the need for coupled stress-strain and damage solutions. This approach eliminates complex nonlinear iterative analyses, thus achieving higher computational efficiency and stability compared to traditional structural damage analysis methods.

**Conclusion**

This paper develops a framework for assessing damage in box-shaped steel members through the establishment of damage factors and the criterion grounded in elastic modulus, facilitating the simulation of damage evolution via stiffness degradation. A novel computational approach for evaluating the load-bearing capacity of compromised steel truss structures is introduced, coupled with an adaptive damage simulation method for ultimate load capacity analysis. The following conclusions can be drawn:

- (1) A HGYF for the box section with the high fitting degree is established through the GYF of GB50017. Based on the definition of EBRs and reference EBRs in HGYF, a damage criterion for box steel components is established to determine whether the component is damaged. The analysis process does not involve the interaction between the stress-strain field and damage, thereby reducing the difficulties caused by coupling calculation.
- (2) The formulation of a damage factor based on the conservation principle of deformation energy allows for a quantitative assessment of damage severity through reductions in elastic modulus. This underpins the

| Steps J | $r_k^0(10^{-4})$ | $r_k^e(10^{-4})$ | $M_{0k}^e$ | $D_{0k}^e$ | $\prod_{i=1}^k M_{0k}^e$ | $D_k^e$ |
|---------|------------------|------------------|------------|------------|--------------------------|---------|
| 2       | 215              | 286              | 0.725      | 0.275      | 0.725                    | 0.275   |
| 3       | 210              | 283              | 0.709      | 0.291      | 0.514                    | 0.486   |
| 4       | 204              | 281              | 0.692      | 0.308      | 0.356                    | 0.644   |
| 5       | 200              | 278              | 0.680      | 0.320      | 0.242                    | 0.758   |
| 6       | 196              | 276              | 0.670      | 0.330      | 0.162                    | 0.838   |
| 7       | 193              | 275              | 0.662      | 0.338      | 0.107                    | 0.893   |
| 8       | 191              | 273              | 0.655      | 0.345      | 0.070                    | 0.930   |
| 9       | 188              | 272              | 0.646      | 0.354      | 0.045                    | 0.955   |
| 10      | 186              | 272              | 0.636      | 0.364      | 0.029                    | 0.971   |
| 11      | 183              | 271              | 0.626      | 0.374      | 0.018                    | 0.982   |
| 12      | 180              | 271              | 0.615      | 0.385      | 0.011                    | 0.989   |
| 13      | 178              | 270              | 0.605      | 0.395      | 0.007                    | 0.993   |
| 14      | 175              | 269              | 0.594      | 0.406      | 0.004                    | 0.996   |
| 15      | 172              | 269              | 0.582      | 0.418      | 0.002                    | 0.998   |
| 16      | 170              | 268              | 0.572      | 0.428      | 0.001                    | 0.999   |
| 17      | 167              | 267              | 0.563      | 0.437      | 0.001                    | 0.999   |
| 18      | 165              | 267              | 0.551      | 0.449      | 0                        | 1       |
| 19      | 162              | 267              | 0.541      | 0.459      | 0                        | 1       |
| 20      | 160              | 266              | 0.531      | 0.469      | 0                        | 1       |
| 21      | 158              | 266              | 0.524      | 0.476      | 0                        | 1       |
| 22      | 157              | 266              | 0.517      | 0.483      | 0                        | 1       |
| 23      | 156              | 266              | 0.511      | 0.489      | 0                        | 1       |
| 24      | 154              | 266              | 0.504      | 0.496      | 0                        | 1       |
| 25      | 153              | 266              | 0.499      | 0.501      | 0                        | 1       |
| 26      | 153              | 266              | 0.497      | 0.503      | 0                        | 1       |
| 27      | 32.72            | 22.15            | 1          | 0          | 1                        | 0       |
| 28      | 32.69            | 22.11            | 1          | 0          | 1                        | 0       |
| 29      | 32.65            | 22.07            | 1          | 0          | 1                        | 0       |
| 30      | 32.62            | 22.03            | 1          | 0          | 1                        | 0       |

**Table 10.** Damage parameters for element 5 (Class 1 of high-stressed element).

- adaptive simulation of damage evolution in steel truss structures via stiffness degradation, obviating the need for intricate coupling calculations associated with stress-strain and damage, and diminishing reliance on finite element mesh discretization.
- (3) The adaptive damage simulation method proposed in this article solves the ultimate bearing capacity of steel truss structures through linear elastic iteration, avoiding the complex process of nonlinear iteration of strain energy calculation through stress-strain. The calculated results are in good agreement with experimental results, ensuring high efficiency and accuracy of structural damage analysis and bearing capacity calculation.



| Steps J | $r_k^0(10^{-4})$ | $r_k^e(10^{-4})$ | $M_{0k}^e$ | $D_{0k}^e$ | $\prod_{i=1}^k M_{0k}^e$ | $D_k^e$ |
|---------|------------------|------------------|------------|------------|--------------------------|---------|
| 2       | 215              | 187              | 1          | 0          | 1                        | 0       |
| 3       | 210              | 188              | 1          | 0          | 1                        | 0       |
| 4       | 204              | 187              | 1          | 0          | 1                        | 0       |
| 5       | 200              | 188              | 1          | 0          | 1                        | 0       |
| 6       | 196              | 190              | 1          | 0          | 1                        | 0       |
| 7       | 193              | 192              | 1          | 0          | 1                        | 0       |
| 8       | 191              | 196              | 0.971      | 0.029      | 0.971                    | 0.029   |
| 9       | 188              | 201              | 0.935      | 0.065      | 0.908                    | 0.092   |
| 10      | 186              | 206              | 0.896      | 0.104      | 0.814                    | 0.186   |
| 11      | 183              | 211              | 0.859      | 0.141      | 0.699                    | 0.301   |
| 12      | 180              | 215              | 0.827      | 0.173      | 0.578                    | 0.422   |
| 13      | 178              | 217              | 0.802      | 0.198      | 0.464                    | 0.536   |
| 14      | 175              | 218              | 0.786      | 0.214      | 0.364                    | 0.636   |
| 15      | 172              | 216              | 0.776      | 0.224      | 0.282                    | 0.718   |
| 16      | 170              | 216              | 0.765      | 0.235      | 0.216                    | 0.784   |
| 17      | 167              | 215              | 0.756      | 0.244      | 0.163                    | 0.837   |
| 18      | 165              | 213              | 0.748      | 0.252      | 0.122                    | 0.878   |
| 19      | 162              | 215              | 0.724      | 0.276      | 0.088                    | 0.912   |
| 20      | 160              | 221              | 0.689      | 0.311      | 0.061                    | 0.939   |
| 21      | 158              | 225              | 0.662      | 0.338      | 0.040                    | 0.960   |
| 22      | 157              | 229              | 0.640      | 0.360      | 0.026                    | 0.974   |
| 23      | 156              | 232              | 0.622      | 0.378      | 0.016                    | 0.984   |
| 24      | 154              | 234              | 0.606      | 0.394      | 0.010                    | 0.990   |
| 25      | 153              | 236              | 0.594      | 0.406      | 0.006                    | 0.994   |
| 26      | 153              | 237              | 0.586      | 0.414      | 0.003                    | 0.997   |
| 27      | 32.72            | 22.15            | 1          | 0          | 1                        | 0       |
| 28      | 32.69            | 22.11            | 1          | 0          | 1                        | 0       |
| 29      | 32.65            | 22.07            | 1          | 0          | 1                        | 0       |
| 30      | 32.62            | 22.03            | 1          | 0          | 1                        | 0       |

Table 11. Damage parameters for element 15 (Class 2 of high-stressed element).

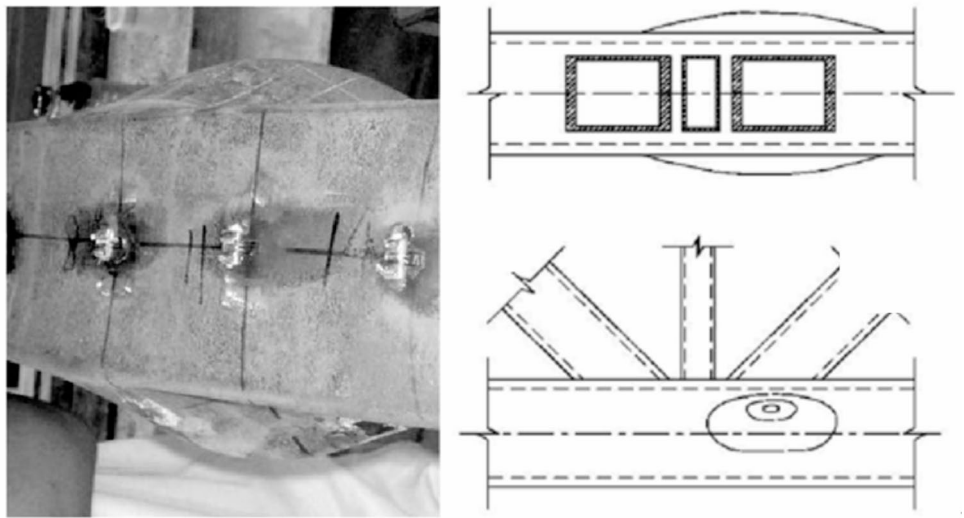


Fig. 8. Schematic diagram of test damage location<sup>39</sup>.

| Experimental results <sup>39</sup> | NE=2   | NE=3   | NE=4   | NE=5   |
|------------------------------------|--------|--------|--------|--------|
| 369.3                              | 370.96 | 367.06 | 368.54 | 371.70 |

**Table 12.** Ultimate bearing capacity/kN.

## Data availability

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

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## Declarations

## Competing interests

The authors declare no competing interests.

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