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Revisit the spin-FET: Multiple reflection, inelastic scattering, and lateral size effects

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We revisit the spin-injected field effect transistor (spin-FET) in a framework of the lattice model by applying the recursive lattice Green's function approach. In the one-dimensional case the results of simulations in coherent regime reveal noticeable differences from the celebrated Datta-Das model, which lead us to an improved treatment with generalized result. The simulations also allow us to address inelastic scattering and lateral confinement effects in the control of spins. These issues are very important in the spin-FET device.

The spin-valve device¹⁻³ and spin-injected field effect transistor (spin-FET)⁴ lie at the heart of spintronics. The basic principle of this type of devices is modulating the resistance by controlling the spins of the carriers^{5,6}, in particular by employing two ferromagnetic (FM) leads as a polarization generator and detector. In practice, there existed two major challenges: (i) spin-polarized injection into a semiconducting channel, and (ii) gate control of the Rashba spin-orbit coupling (SOC) in the channel. The former difficulty has been largely overcome through efforts of many groups⁷⁻¹⁰. For the latter issue, investigations included the gate-voltage-control of the spin precession in both the quantum wells¹¹ and quantum wires¹², and some detailed studies such as the multichannel mixing effects (lateral size effects)¹³⁻²⁰. An integration of the ingredients of the two types mentioned above into a single device using AsIn heterostructure with a top gate, was realized in a recent experiment²¹. It is remarkable that this progress has renewed an interest in the spin-FET device^{18,20,22-24}, which has been proposed by Datta and Das more than two decades ago⁴.

In this work we revisit this novel spintronic device, via simulations on a quantum-wire model (semiconductor nanowire implementation), based on the powerful recursive lattice Green's function (GF) technique. To reach realistic scales, for the InSb material parameters (with large Landé g factor and strong spin-orbit coupling²⁵), we design our simulation for a quantum wire (about 300 nm length in longitudinal direction) with 500 lattice sites. We may summarize the present study by the following achievements: (i) In the ideal one-dimensional (1-D) case and coherent regime, the simulation of the energy-resolved transmission spectrum and the SOC-modulation of the transmission peak reveal differences from the well-known Datta-Das model⁴. Accordingly, we develop a Fabry-Perot cavity model to obtain an analytic result which generalizes its counterpart in Ref. 4, and a more recent work²³. (ii) The employed recursive GF technique allows us for an efficient simulation of decoherence effect in spatial-motion which, quite *indirectly*, degrades the control of the spin precession. It is of particular interest that this treatment does not involve any explicit spin-flip mechanisms^{26,27}, but only incorporates the Büttiker phasebreaking model²⁸⁻³² to introduce spatial decoherence effect. The result of simulation agrees with the temperature dependence observed in experiment²¹, and substantiates the mesoscopic (coherence) requirement mentioned in the Datta-Das proposal⁴ or the non-diffusive (ballistic) criterion²⁴. (iii) We simulate the effect of lateral confinement by setting 20 and 40 lattice sites for the width of the quantum wire. The results are in consistence with some previous studies of continuous wave-guide models¹³⁻²⁰, implying that the lateral size will influence the functionality of the spin-FET device, if it exceeds a certain range (drastically violating the 1-D condition).

Results

The device consists of a quantum wire (central region) and of two FM leads, described by total Hamiltonian $H = H_w + \sum_{\beta = L,R} H_\beta + H_T$, with

$$H_{\rm w} = \sum_{i=1}^{M} \epsilon_0 c_i^{\dagger} c_i - \sum_{i=1}^{M-1} \left[\left(t_0 c_{i+1}^{\dagger} c_i - i \alpha c_{i+1}^{\dagger} \sigma_y c_i \right) + \text{H.c.} \right], \tag{1a}$$

$$H_{\beta} = \sum_{i=1}^{\infty} b_{\beta,i}^{\dagger} (\epsilon_{\beta} + \boldsymbol{\sigma} \cdot \boldsymbol{h}_{\beta}) b_{\beta,i} - \sum_{i=1}^{\infty} (t_{\beta} b_{\beta,i+1}^{\dagger} b_{\beta,i} + \text{H.c.}),$$
(1b)

$$H_T = -t_c \left[\left(c_1^{\dagger} b_{L,1} + c_M^{\dagger} b_{R,1} \right) + \text{H.c.} \right].$$
(1c)

For the sake of simplicity, here we have assumed a 1-D tightbinding lattice model (with M lattice sites for the quantum wire). For a narrow SOC channel (the working area of the spin-FET), the inter-subband mixing effect is negligible. In this case the 1-D model is applicable⁴. The electronic creation and annihilation operators are abbreviated by a vector form, e.g., $c_i^{\dagger} = \left(c_{i\uparrow}^{\dagger}, c_{i\downarrow}^{\dagger}\right)$ and $b_{\beta,i}^{\dagger} = \left(b_{\beta,i\uparrow}^{\dagger}, b_{\beta,i\downarrow}^{\dagger} \right)$, where *i* labels the lattice site and (\uparrow, \downarrow) the spin orientations. The Pauli matrices are introduced as $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. In the quantum wire Hamiltonian (H_w), ϵ_0 and t_0 are the tight-binding site energy and hopping amplitude; α is the SOC strength. Notice that for a continuous limit, the corresponding SOC strength should be $\tilde{\alpha} = 2a\alpha$, where *a* is the lattice constant. For the FM leads (H_{β}) , ϵ_{β} , t_{β} , and h_{β} are, respectively, the tight-binding parameters and the FM exchange field. For the spin-FET, we assume the FM leads magnetized in parallel, with $h_{\beta} = h_0(0, 0, 1)$ for *z*-axis magnetization. For the wire-lead coupling (H_T) , we assume a common coupling amplitude t_c at both sides.

In our simulation, for the central quantum wire, we refer to the SOC strength of the InSb material, $\tilde{\alpha} = 0.2 \text{ eV} \cdot \text{Å}$. This implies a SOC length $l_{so} \simeq 200 \text{ nm}$. Assuming a lattice constant $a \simeq 6 \text{ Å}$, we then decide to simulate the 1-D quantum wire with M = 500 lattice sites (length of $\sim 300 \text{ nm}$), in order to be longer than l_{so} for the purpose of spin-FET. Finally, we assume $t_0 = 1.0 \text{ eV}$, the splitting exchange energy $h_0 = 0.4 \text{ eV}$, and $t_c = 0.4t_0$. For the FM leads and the fictitious side-chains (to be addressed later), we set $t_{\beta} = t_I = 0.8 \text{ eV}$.

Transmission Spectrum and SOC-modulation Lineshape. Let us consider first a *coherent* transport through the quantum wire (corresponding to the case of low temperatures²¹). In Fig. 1(a), we display the representative results of the transmission spectrum and observe clear SOC(α)-modulation effect. Owing to finite length of the quantum wire, the transmission spectrum reveals the usual peakversus-valley structure. However, the SOC interaction would alter the location of the energy levels, and cause level splitting as revealed by the additional fine-structures (see, for instance, the red curve). Viewing this complexity of energy levels under the SOC influence, we employ the height of the transmission peak to characterize the modulation effect. The extracted results are shown in Fig. 1(b). Note that for the lattice system under study, for a given SOC α , the transmission peaks at distant energies may have different heights. To capture this feature, we have chosen the peaks from several relatively distant energy intervals.



Figure 1 | (a) SOC(α)-modulation effect on the transmission spectrum, with modulation period $\alpha^* = \pi t_0/M$ (see the main text for more detail). In particular, at $\alpha = 0.5\alpha^*$ the entire transmission spectrum is suppressed, indicating an *off*-state of the spin-FET. (b) SOC(α)-modulation to the heights of the transmission peaks. Illustrative results are shown for three energy intervals: curve I for $E/t_0 \in (1.305, 1.315)$, II for $E/t_0 \in (1.45, 1.46)$, and III for $E/t_0 \in (1.70, 1.71)$.

We find that the SOC-modulation period is well described by $\alpha^* = \pi t_0/M$, where M = L/a. This is in perfect agreement with the result from a simple plane-wave-based interference analysis. Following Ref. 4, the phase difference caused by the SOC over distance *L* between the spin-up and spin-down components is given by $\theta = (2m^*/\hbar^2)\tilde{\alpha}L$. Converting to the lattice model via the replacement $\hbar^2/2m^* \rightarrow t_0 a^2$, the above α^* is obtained by the condition $\theta = 2\pi$ (note that $\tilde{\alpha} = 2a\alpha$).

However, the SOC-modulation lineshape does not coincide in general with the prediction of the Datta-Das model⁴. It was remarked in Ref. 4, that the spin precession in the device and the SOC-modulated transmission are independent of the incident energy. However, as we will prove shortly, this is not true. Also, we find different transition behaviors around the (modulation) peaks and valleys: the variation around the peak can be much slower (forming almost a "plateau") than the change around the valley. Below we present a semi-quantitative analysis based essentially on the same Datta-Das model but accounting for *multiple reflections* in the SOC region, which generalizes the central result in Ref. 4. We notice also that this type of multiple reflections has not been taken into account when fitting and analyzing the experimental result^{21,23,24}.

Semi-quantitative Analysis. Let us consider a 1-D continuous model for the quantum wire embedded in between two FM leads. This is similar to an optical two-sided Fabry-Perot cavity system³³, with the electron transmission as an analog of the optical wave in terms of the so-called Feynman paths³⁴. Of particular interest in the electronic setup of spin-FET is the SOC-modulation in the "cavity", which is described by the continuous version of the Rashba model as $H_{\rm so} = \tilde{\alpha} (\sigma_x k_y - \sigma_y k_x) \equiv -\tilde{\alpha} k \sigma_y$, owing to the 1-D motion with $k_y = 0$.

According to Datta and Das⁴, a single passage through the SOC-wire will result in a spin precession with angle $\theta = (k_{+} - k_{-})L = (2m^*/\hbar^2)\tilde{\alpha}L$, where k_{\pm} are given by the solution

from $E = \hbar^2 k^2 / 2m^* \mp \tilde{\alpha}k$, for a given energy *E*. (For an 1-D lattice model, similar consideration also gives that the spin precession angle θ is independent of the energy). Then, taking into account the role of the FM leads, a transmission coefficient was proposed in Ref. 4 as $T \propto \cos^2(\theta/2)$. In practice²¹, this result has been applied to analyze experiment by fitting the measured *nonlocal* voltage with $V = A \cos(2m^*\tilde{\alpha}L/\hbar^2 + \varphi)$. In the subsequent studies^{23,24}, deeper analysis was carried out for the fitting parameters (amplitude *A* and phase φ), and some aspects of the experiment were explained while some others remaining unclear²³.

A drawback in the above treatment is the neglect of the (infinite) multiple reflections, which should exist in any two-leads connected electronic devices. We then perform a Fabry-Perot-cavity type analysis to account for the multiple reflections (see the "Methods" part for more details), and obtain the transmission coefficient as

$$T = \frac{4t^4 \cos^2(\theta/2) \sin^2(KL)}{D^2 + 4t^4 \cos^2(\theta/2) \sin^2(KL)}.$$
 (2)

In this result we have denoted $(r-1)^2 \sin^2(\theta/2) + 4r \sin^2(KL) \equiv D$ and $(k_+ + k_-)L/2 \equiv KL$. For the contact of the quantum wire with the FM leads, we assumed identical transmission (*t*) and reflection (*r*) amplitudes at the two sides. Note that after accounting for the multiple reflections, Eq. (2) generalizes the result of Ref. 4, in an elegant way.

Based on Eq. (2) we show in Fig. 2 the SOC-modulation effect on the (resonant) transmission peak. Interestingly, we find similar line-shape as in Fig. 1(b). In particular, different transition behaviors are found around the peak (maximum) and dip (minimum), for the case $t \simeq 1$. By setting t = 1 we obtain from Eq. (2) that $T_p = 4 \cos^2(\theta/2)/[1 + \cos^2(\theta/2)]^2$. This result, in a simple way, allows us to explain the "plateau" behavior of $t \simeq 1$ in Fig. 2.

Using Eq. (2), one may qualitatively understand the modulation behavior in Fig. 1(b). Compared the lattice system described by Eq. (1) with the Fabry-Perot cavity model, an obvious difference is that the former does not have a constant single-side transmission (t) and reflection (r), where the effective t and r should depend on the energy (E) and the SOC α . Therefore, from Eq. (2), the transmission peak T_p may have different height in different energy region and may depend on α through the effective t and r. In addition to the multiple



Figure 2 | SOC(α)-modulation effect from a Fabry-Perot-type resonator model consideration, in which the multiple reflections are essentially accommodated. The (resonant) transmission peak (T_p) is obtained from Eq. (2) and a "plateau" behavior is recovered when the single-side transmission is nearly transparent (the transmission coefficient $t \simeq 1$).

reflections, this should be the reason that lead to the non-overlapped modulation lineshapes in different energy areas and the "plateau" behavior around the modulation peak, as shown in Fig. 1(b).

Decoherence Effect. We apply the Büttiker fictitious reservoir approach to account for the possible inelastic scattering in the device^{28–32}. This phenomenological approach is very efficient in comparison with any other microscopic model based treatments. In Fig. 3 we show the decoherence effect on the SOC-modulation. Qualitatively speaking, the inelastic scatterers would cause a large number of forward and backward propagation pathways. Simple analysis in terms of the time-reversal symmetry tells us that the forward and backward propagation over equal distance would cancel the spin precession. As a result, for any transmitted electron (from the left to the right leads), the *net* distance of spin precession is the length of the quantum wire. This explains the common SOC-modulation period (α^*) in Fig. 3 when altering the inelastic scattering strength (η).

However, the SOC-modulation amplitude will be suppressed by enhancing the inelastic scattering strength. The fictitious reservoir model is very convenient to account for phase breaking (decoherence) of spatial motion, through destroying quantum interference between partial waves. Nevertheless, the role of this model is not so straightforward for the SOC caused spin precession. We may remark that in our treatment we did not introduce the explicit spin-relaxation mechanism^{26,27}, whose effect is relatively more direct^{23,24}. In Ref. 4, Datta and Das pointed out that, in order to perform the spin-FET, one of the essential requirements is the central conducting channel within a mesoscopic phase-coherent regime. Our result in Fig. 3 substantiates this requirement, and a general statement that the Rashba-spin-control does not work in diffusive transport regime²⁴. The present result is also in agreement with the experiment²¹, where the SOC modulation effect was found to be washed out by stronger inelastic scattering (more phonon excitations) by increasing the temperatures.

Lateral-Size Effect. In the original proposal the intersubband coupling effect owing to lateral size was excluded for a narrow (quasi-1D) quantum wire⁴. Below, we consider the lateral-size effect by simulating a quasi-two-dimensional (2D) quantum ribbon with $M \times N$ lattice sites. Accordingly, we need to generalize each lattice site of the 1-D wire to a lateral column with N sites along the *y*-direction. While the 2D generalization of the tight-binding



Figure 3 | Decoherence effect on the SOC(α)-modulation displayed in Fig. 1(b), for the transmission peak (T_p extracted from $\mathcal{T}_{eff}(\epsilon)$) with $E/t_0 \in (1.305, 1.315)$. The coupling coefficient (η) to the fictitious sidereservoirs characterizes well the decoherence strength in the Büttiker phase-breaking approach.

model is straightforward, we only specify the SOC Hamiltonian in 2D case as

$$H_{SO} = \sum_{i} \left[i\alpha \left(a_{i}^{\dagger} \sigma_{y} a_{i+\delta_{x}} - a_{i}^{\dagger} \sigma_{x} a_{i+\delta_{y}} \right) + \text{H.c.} \right].$$
(3)

Here, the summation is over the $M \times N$ lattice sites, and $(\delta_x, \delta_y) = (1, 1)$ denote displacements over a unit lattice cell along the longitudinal (x) and lateral (y) directions.

In Fig. 4 we show the effects of the lateral size (with N = 20 and 40). First, owing to the energy sub-bands (and their mixing) caused by the lateral confinement, the transmission peak can exceed unity (in the 1-D case the maximal T_p is unity). One may notice that, unlike prediction from the standard Landauer-Büttiker formula, the height of the transmission peak, which is proportional to the differential conductance at the corresponding energy (bias), does not equal the lateral-channel numbers involved. In 1-D case, the transmission peak is originated from a constructive interference given by the standing-wave condition of the longitudinal wave-vector. This resonant condition (together with symmetric coupling to the leads) will result in a transmission coefficient of unity. However, for a given energy (E) in the quantum-ribbon system, different lateral channels are associated with different longitudinal wave-vectors. Then, the resonant condition for each transverse channel cannot be satisfied simultaneously.

The second effect originated from the lateral motion is the SOCinduced *additional* spin precession. In general, this will affect the SOC-modulation quality of the spin-FET. For small *N* (with respect to the SOC length with ~300 sites in the present study), this effect is not prominent (see, for instance, the result of N = 20 in Fig. 4). However, with the increase of the lateral size, the transmission cannot be switched off (particularly at higher α), as illustrated by the result of N = 40. Moreover, for even larger lateral-size and SOC- α , or in some energy domain, the transmission modulation will become strongly irregular (not shown in Fig. 4). We then conclude that, while the longitudinal modulation period (α^*) keeps unchanged, the quality of the spin-FET performance will be degraded with the increase of the lateral size. Only for narrow quantum wire (small *N*) one can define desirable working region for the spin-FET. This remark supports the conclusion in Ref. 18, and some previous studies¹³⁻¹⁷. It



Figure 4 | Effect of the confined lateral motion on the SOC(α)modulation. Within the lattice model, in addition to the longitudinal number of sites M = 500, we set N = 20 and 40 to reveal the increasingly non-negligible lateral size effect (The imperfect result of N = 40 is owing to the not high enough numerical accuracy of determination for the extremely dense transmission peaks).

seems that an exception is the 2D system with semi-infinite (considerably wide) width, where the SOC modulation effect can be restored, despite of the degraded quality^{19,20,23,24}.

Discussion

In addition to the simulated decoherence and lateral-size effects, a key result of this work is Eq. (2) which generalizes the Datta-Das analysis^{4,21,23}. Fig. 2 shows only the SOC-modulation effect on the transmission peak. For the entire transmission spectrum, given by Eq. (2), it is clear that the SOC-modulated transmission depends on the incident energy, through the K-dependence. This will more dramatically affect the finite-bias current through the spin-FET, compared with the *energy-independent* modulated transmission⁴. It was highlighted in Ref. 4, that the energy-independent modulation "property", observed from the differential phase shift $\theta = 2m^* \tilde{\alpha} L/\hbar^2$, implies an important advantage for quantum-interference device applications. That is, it can avoid washing out the interference effects and achieve large percentage modulation of the current, even in the multimoded devices operated at elevated temperatures and large applied bias⁴. It seems of interest to perform further examinations on these issues based on Eq. (2).

To summarize, we have revisited the transport rooted in the spin-FET device, with the help of the powerful recursive lattice Green's function approach. Our result of the energy-resolved transmission spectrum reveals noticeable differences from the Datta-Das model⁴, which motivated us to develop a Fabry-Perot-cavity type treatment to generalize the central result. We also simulated the decoherence and lateral-size effects. The former substantiates the mesoscopic (coherence) requirement⁴ or the non-diffusive (ballistic) criterion²⁴, and it is in reasonable agreement with the observation in the recent experiment²¹. The latter implies additional restrictions to the Rashba-spin-control and thus the quality of the device.

Methods

Inelastic Scattering Model. The basic idea of the phenomenological phase-breaking approach proposed by Büttiker^{28,29} is to attach the system (quantum wire) to some additional *virtual* electronic reservoirs. The transport electron is assumed to partially enter the virtual reservoir, suffer an inelastic scattering in it (then lose the phase information), and return back into the system (to guarantee the conservation of electron numbers). Technically, we model the virtual reservoir (coupled to the $J_{\rm th}$ site of the quantum wire) by a tight-binding chain with Hamiltonian^{30–32}

$$\tilde{H}_{J} = \sum_{i=1}^{\infty} \epsilon_{0} b_{J,i}^{\dagger} b_{J,i} - \sum_{i=1}^{\infty} \left(t_{J} b_{J,i+1}^{\dagger} b_{J,i} + \text{H.c.} \right),$$
(4)

and this chain is coupled to the quantum wire through a coupling Hamiltonian

$$\tilde{H}_{T,J} = -\left(\eta c_J^{\dagger} b_{J,1} + \text{H.c.}\right),\tag{5}$$

with η the coupling strength. In this work, for the quantum wire with M = 500 (length of ~300 nm), we assume to attach 10 side-reservoirs (so $J = 50,100, \dots, 500$), which represent a mean-distance of $50a \simeq 30$ nm between the nearest-neighbor inelastic scatterers.

Lattice Green's Function Approach. Applying the powerful lattice Green's function method, in particular combining with a *recursive* algorithm³⁴, one can calculate the retarded and advanced Green's functions and obtain the transmission coefficients between any pair of leads (reservoirs) as follows³⁴⁻³⁶:

$$T_{\mu\nu}(\epsilon) = \operatorname{Tr} \left[\Gamma_{\mu}(\epsilon) G^{r}(\epsilon) \Gamma_{\nu}(\epsilon) G^{a}(\epsilon) \right].$$
(6)

Here μ and ν denote all the reservoirs, including the left and right leads together with the virtual inelastic scattering reservoirs. Formally, $\Gamma_{\mu}(\epsilon) = i \left[\sum_{\mu}^{r}(\epsilon) - \sum_{\mu}^{a}(\epsilon)\right]$, and $G^{r}(\epsilon) = \left[G^{a}(\epsilon)\right]^{\dagger} = 1 / \left[\epsilon - H_{\rm w} - \sum_{\mu=L,R,\{J\}} \left(\Sigma_{\mu}^{r}\right)\right]$. $\Sigma_{\mu}^{r(a)}$ is the retarded (advanced) self-energy owing to coupling with the $\mu_{\rm th}$ lead (reservoir). Knowing $T_{\mu\nu}$, the entire *effective transmission coefficient* from the left to the right lead can be straightforwardly obtained through³⁰⁻³²

$$\mathcal{T}_{\rm eff}(\epsilon) = T_{LR} + \sum_{\mu,\nu=1}^{N} K_{\mu}^{(L)} W_{\mu\nu}^{-1} K_{\nu}^{(R)}.$$
(7)

Here, $K_{\mu}^{(L)} = T_{L\mu}$ and $K_{\nu}^{(R)} = T_{\nu R}$. W^{-1} is the inverse of the matrix W with elements $W_{\mu\nu} = (1 - R_{\nu\nu})\delta_{\mu\nu} - T_{\mu\nu}(1 - \delta_{\mu\nu})$, where $R_{\nu\nu} = 1 - \sum_{\mu(\neq\nu)} T_{\nu\mu}$. Inserting the transmission coefficient $\mathcal{T}_{eff}(\epsilon)$ into the Landauer-type formula, one can easily compute the transport current. In this work, however, we will simply use $\mathcal{T}_{eff}(\epsilon)$ (corresponding to differential conductance) to characterize the modulation effects in the spin-FET.

Multiple Reflections. Following the Fabry-Perot cavity type model^{33,34}, we consider the transmission and reflection of an electron at the right FM lead, which entered from the left FM lead with a wave function (after passing through the left contact junction):

$$|\psi_1\rangle = a|\uparrow\rangle_y + b|\downarrow\rangle_y. \tag{8}$$

At the right side of the 1-D wire (cavity), after single passage (over L) under the SOC influence, the electron state evolves to

$$\begin{split} |\psi_{2}\rangle &= a e^{ik_{+}L} |\uparrow\rangle_{y} + b e^{ik_{-}L} |\downarrow\rangle_{y} \\ &= \begin{pmatrix} e^{ik_{+}L} & 0 \\ 0 & e^{ik_{-}L} \end{pmatrix} |\psi_{1}\rangle \end{split}$$
(9)

Here and in the following, using the transfer matrix representation, the states should be understood as column vectors in the basis $\{|\uparrow\rangle_{y}, |\downarrow\rangle_{y}\}$, e.g., $|\psi_{1}\rangle = (a, b)^{T}$. For a given energy E, k_{\pm} are solved from $E = \hbar^{2}k^{2}/2m^{*} \mp \tilde{a}k$, corresponding to the momentums of the spin-up and spin-down electrons.

Since the FM leads are polarized in the *z*-direction, at the right side, only the electron with spin state $|\uparrow\rangle_z$ can enter the right lead (with transmission amplitude *t* and reflection amplitude *r*). For electron with $|\downarrow\rangle_z$, it will be *fully* reflected. Accordingly, based on $|\psi_2\rangle$, the transmitted wave into the right lead is given by

$$|\psi\rangle_R^{(1)} = t\hat{P}_{z\uparrow}|\psi_2\rangle \equiv U_R|\psi_1\rangle.$$
(10)

In this context we introduce the projection operator

$$\hat{P}_{z\uparrow} = |\uparrow\rangle_z \langle\uparrow| = \frac{1}{2} \begin{pmatrix} 1 & 1\\ 1 & 1 \end{pmatrix}, \tag{11}$$

and similarly

$$\hat{P}_{z\downarrow} = |\downarrow\rangle_z \langle\downarrow| = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix},$$
(12)

for the following use. In Eq. (10) we also defined a transfer matrix which reads

$$U_R = \frac{t}{2} \begin{pmatrix} e^{ik_+L} & e^{ik_-L} \\ e^{ik_+L} & e^{ik_-L} \end{pmatrix}.$$
 (13)

At the same time, the reflected wave from the right junction is given by

$$\left|\tilde{\psi}\right\rangle_{R}^{(1)} = \hat{P}_{z\downarrow}|\psi_{2}\rangle + r\hat{P}_{z\uparrow}|\psi_{2}\rangle \equiv \tilde{U}_{R}|\psi_{1}\rangle, \tag{14}$$

where

$$\tilde{U}_{R} = \frac{1}{2} \begin{bmatrix} (r+1)e^{ik_{+}L} & (r-1)e^{ik_{-}L} \\ (r-1)e^{ik_{+}L} & (r+1)e^{ik_{-}L} \end{bmatrix}.$$
 (15)

Similar analysis gives the transfer matrix acting on the wave inversely propagated from the right side to the left one and reflected at the left junction:

$$\tilde{U}_L = \frac{1}{2} \begin{bmatrix} (r+1)e^{ik_-L} & (r-1)e^{ik_+L} \\ (r-1)e^{ik_-L} & (r+1)e^{ik_+L} \end{bmatrix}.$$
(16)

Therefor, the total wave arriving to the right FM lead is a sum of all the partial waves, given by

$$\begin{split} |\Psi\rangle_{R} &= |\psi\rangle_{R}^{(1)} + |\psi\rangle_{R}^{(2)} + |\psi\rangle_{R}^{(3)} + \cdots \\ &= \left(U_{R} + U_{R}\tilde{U}_{L}\tilde{U}_{R} + \cdots\right)|\psi_{1}\rangle \\ &= U_{R}\left(1 - \tilde{U}_{L}\tilde{U}_{R}\right)^{-1}|\psi_{1}\rangle. \end{split}$$
(17)

Noting that $|\psi_1\rangle = t|\uparrow\rangle_z$, we finally obtain the total transmission probability as

$$T = |_{z} \langle \uparrow | \Psi \rangle_{R} |^{2} = \frac{4t^{4} \cos^{2}(\theta/2) \sin^{2} KL}{D^{2} + 4t^{4} \cos^{2}(\theta/2) \sin^{2} KL}.$$
 (18)

Here we defined $KL = (k_+ + k_-)L/2$ and $\theta = (k_+ - k_-)L$, and introduced $D = (r - 1)^2 \sin^2(\theta/2) + 4r \sin^2 KL$.

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Author contributions

X.Q.L initiated the idea and supervised the work. L.T.X carried out all the analytic derivations and numerical calculations under technical supports from Q.F.S. X.Q.L wrote the paper and all the authors reviewed it.

Additional information

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