PHASES OF SPONTANEOUS STRINGS IN LARGE DIMENSIONS ☆

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We point out the existence of different phases for strings with extrinsic curvature stiffness. These phases form at low tension in the limit of infinite spatial dimension.

Strings with an extrinsic curvature stiffness, as proposed recently by Polyakov [1] and the author [2][#1], are apparently a much better idealization of the strings which bind quarks in QCD than the ordinary Nambu–Goto strings. In the absence of a tension term, the classical action is scale invariant, just as in QCD. In the limit of infinite dimension d it can be shown that fluctuations generate a mass scale. It plays a similar role as the dimensionally transmuted coupling constant in QCD. It is observed as a spontaneously generated tension giving a confining potential between quarks [4,5]. Moreover, contrary to the ordinary string [6], the potential has the desired asymptotically free $1/R$ behaviour at short distances [5,7]. At higher temperature, the tension disappears in a thermal deconfinement transition [8]. Recently it has been shown [9,10], always in the limit $d\to\infty$, that if such a "spontaneous string" is supplemented with an additional Nambu–Goto term of a certain (negative) critical size, it develops an instability. The purpose of this note is to study the phases which may be expected in this regime.

The action of the mixed string is

$$\begin{aligned}\mathscr{A} &= \mathscr{A}_{\rm NG} + \mathscr{A}_{\rm K} \\ &= M^2_{\rm NG}\int d^2\xi\,\sqrt{g} \\ &\quad + \frac{1}{2\alpha}\int d^2\xi[\sqrt{g}\,D^2x^a D^2 x^a \\ &\quad + \lambda^{ij}\partial_i x^a \partial_j x^a - \lambda^{ij}(g_{ij}-\delta_{ij})] \\ &= \mathscr{A}_{\rm K0} + \tfrac{1}{2}d\,{\rm tr}\ln(D^4 + g^{-1/2}\partial_i\lambda^{ij}\partial_j)\,, \end{aligned} \tag{1}$$

where $\tilde\alpha = (d/2)\alpha$ and $\mathscr{A}_{\rm K0}$ is the curvature action for the zero-momentum configurations x_0 which are not included in the path integration and which may carry a constant strain (with $x_0 \propto \xi$)

We shall look at planar surfaces in the conformal gauge $g=\rho\delta_{ij}$ and $x_0=\xi^1$, $x_0^a=0$ ($a=2,...,d$) with all fluctuating fields periodic in the interval $\xi^1\in(0,\beta_{\rm ext})$, $\xi^2\in(0,R_{\rm ext})$. For $d\to\infty$, we can go to the saddle point which, on symmetry grounds, has a space independent ρ and $\lambda^{ij}=\lambda\delta^{ij}$. Renormalization introduces a dimensionally transmuted coupling constant $\bar\lambda$ (dimension = mass^2) in terms of which the extremal action becomes

$$\mathscr{A} = -\tfrac{1}{2}dR_{\rm ext}\beta_{\rm ext}g\,, \tag{2}$$

with

$$\begin{aligned} g = \rho\{&(\bar\lambda_\nu/4\pi)\nu - (\lambda/4\pi)[\ln(\lambda/\bar\lambda)-1] \\ &-\lambda/4\pi + \lambda/\tilde\alpha\rho\}\,, \end{aligned} \tag{3}$$

where $\bar\lambda_\nu \equiv \bar\lambda\exp(\nu)$ and ν defined so that $M^2_{\rm NG} = (d/2)\bar\lambda_\nu\nu/4\pi$. This is to be maximized in λ and

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[#1] Our proposal was instigated by previous work on biomembranes by ref. [3].

minimized in ρ, with the result $\lambda=\bar{\lambda}_\nu$, $\rho=\bar{\rho}=4\pi/\bar{\alpha}$. The total tension is $M^2_{tot}=(d/2)\bar{\lambda}_\nu(1+\nu)/4\pi$.

For $\nu=0$ we shall call the string *purely spontaneous*. For $\nu\to-1$, the string becomes critical (the intrinsic area diverges with $\rho\sim1/(1+\nu)$ at fixed M^2_{tot}). The tangent vectors of the surface fluctuate with a correlation length $\xi=1/m^2=1/\bar{\lambda}_\nu\bar{\rho}$. This is the DeGennes persistence length [11]. The calculations by David, Guitter, and Pisarski in refs. [9,10] have provided us with the quadratic variation of g

$$\delta^2 g=(\bar{\lambda}_\nu\bar{\rho}/4\pi)v^{\mathrm{T}}(\xi)M(-\mathrm{i}\partial)v(\xi)\ ,$$
$$v(\xi)\equiv(\delta\rho/\bar{\rho},\delta\lambda^{11}/\bar{\lambda}_\nu,\lambda^{22}/\bar{\lambda}_\nu,\lambda^{12}/\bar{\lambda}_\nu)(\xi)\ , \qquad (4)$$

with the matrix

$$M=\begin{pmatrix} G & -\mathrm{i}E & -\mathrm{i}F & 0\\ -\mathrm{i}E & A & B & 0\\ -\mathrm{i}F & B & C & 0\\ 0 & 0 & 0 & 0\end{pmatrix}, \qquad (5)$$

and, for small $P^2\equiv p^2/m^2$,

$$G(p)\approx-\tfrac{1}{6}P^2-\tfrac{1}{30}P^4+\dots\ ,$$
$$A(p)\approx\tfrac{3}{8}+\tfrac{1}{24}P^2+\tfrac{1}{8}P^2L_0-\tfrac{1}{6}P^4+\dots\ ,$$
$$B(p)\approx\tfrac{1}{8}+\tfrac{1}{24}P^2-\tfrac{1}{12}P^4+\dots\ ,$$
$$C(p)\approx\tfrac{3}{8}+\tfrac{1}{8}P^2+\tfrac{1}{8}P^2L_0-\tfrac{1}{20}P^4+\dots\ ,$$
$$D(p)\approx\tfrac{1}{8}-\tfrac{1}{24}P^2+\tfrac{1}{48}P^4+\dots\ ,$$
$$E(p)\approx\tfrac{1}{2}(1+\nu)+\tfrac{1}{4}P^2-\tfrac{1}{24}P^4+\dots\ ,$$
$$F(p)\approx\tfrac{1}{2}(1+\nu)+\tfrac{1}{12}P^2-\tfrac{1}{120}P^4+\dots\ ,$$
$$L_0(p)\equiv\ln(P^2)\ . \qquad (6)$$

For $\nu<\nu_c\approx-0.902$, the upper 3×3 part of this matrix develops a negative eigenvalue, ε_3. Placing all stable variables at their extremum, the unstable mode can be described approximately in terms of an effective lagrangian involving only $\delta\rho/\bar{\rho}(\xi)$ which we shall call $\varphi(\xi)$. We have verified that the polarization properties of λ^{ij} are irrelevant to the phenomenon by checking that the instability occurs also if we restrict λ^{ij} to its trace (the unstable regime is only very slightly shifted in p, see fig. 1). We can therefore express the variation $\delta^2 g$ as $\bar{\lambda}_\nu\bar{\rho}/4\pi$ times $1+\nu+\mathscr{L}$ with an effective lagrangian

$$\mathscr{L}\equiv\tfrac{1}{2}\varphi(\xi)\,\mathscr{O}(-\mathrm{i}\partial)\varphi(\xi)-\tfrac{1}{3}c\varphi^3(\xi)+\tfrac{1}{4}d\varphi^4(\xi)+\dots\ , \qquad (7)$$

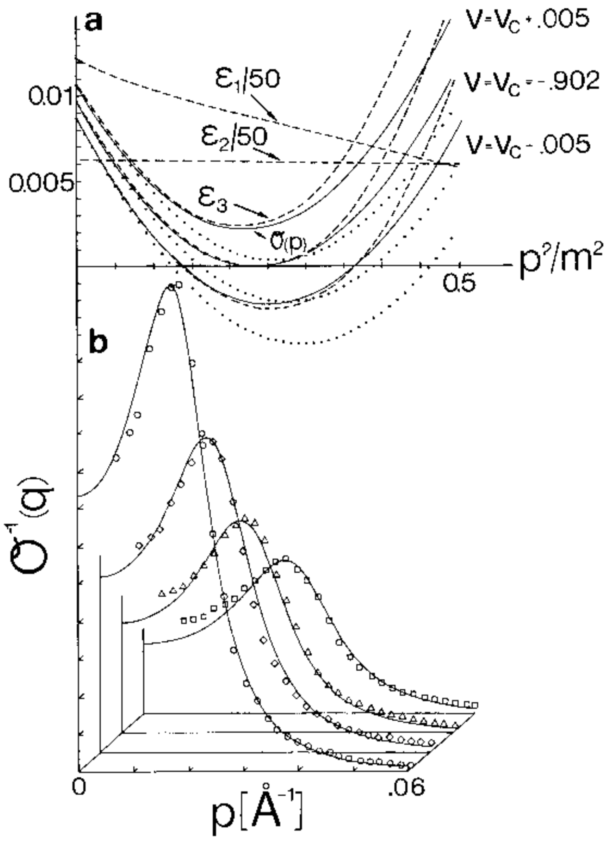

Fig. 1. (a) The kinetic term (8) of the lagrangian (7) for various $\nu\approx\nu_c$. Also plotted are the eigenvalues of the upper 3×3 part of the matrix M (dashed) and the function $G+(A+2B+C)^{-1}\times(E+F)^2$ (dotted). It governs the $\delta\rho/\bar{\rho}$ fluctuations if λ^{ij} is restricted to its trace (showing that the phase transition appears in the trace part of λ^{ii} alone). (b) The structure factor $S(p)$ of D_2O–toluene–butanol–SDS–NaCl in the three-phase regime [12] and comparison with $\mathscr{O}(p)^{-1}$ of the spontaneous string (solid curves) for $\nu\gtrsim\nu_c$.

where

$$\mathscr{O}(p)=G(p)+(E,F)\begin{pmatrix}A & B\\ B & C\end{pmatrix}^{-1}(E,F)$$
$$=\det M\Big/\left[\mathrm{D}\cdot\det\begin{pmatrix}A & B\\ B & C\end{pmatrix}\right]$$
$$\approx a_1[p^2/m^2-p_0^2/m^2-3(\nu-\nu_c)]^2+a\ , \qquad (8)$$

with $p_0^2/m^2\approx0.237$, $a\equiv a_0(\nu-\nu_c)\approx0.45(\nu-\nu_c)$, $a_1\approx0.171$, and c, d being numbers of order unity. We have omitted higher powers in φ and checked that c is sufficiently small to justify such a truncation à la Landau near the instability.

By a rescaling of φ and the total $\mathscr{L}$ (dropping the overall factor in $\mathscr{L}$) we can make $c=1$, $d=1$ and write

$$\mathscr{L}=\tfrac{1}{2}\varphi\{a_0(\nu-\nu_c)+a_1[-\partial^2/m^2-p_0^2/m^2 -3(\nu-\nu_c)]^2\}\varphi-\tfrac{1}{3}\varphi^3+\tfrac{1}{4}\varphi^4\ , \qquad (9)$$

with a_0 of the order unity and $a_1/a_0\approx 0.38$.

For $\nu \gg \nu_c$, the φ fluctuations of a surface can be visualized in scattering experiments, where one sees a structure factor $S(p)\propto \mathscr{O}(p)^{-1}$. A behaviour of precisely this type has recently been observed in small-angle X-ray data of microemulsions [12] (see fig. 1). There [13], the three-phase regime supposedly consists of soap interfaces with small surface tension. If further soap is added, the microemulsion undergoes a phase transition into a great variety of liquid crystal phases. It is therefore interesting to see whether similar things happen to the surface under consideration upon lowering ν towards the critical value ν_c. At the mean-field level, the main candidates for the lowest ground state are given by [14,15]

$$\varphi(\xi)=\varphi_n\left(\sum_{i=1}^{n}\exp[\mathrm{i}(k_i\xi+\alpha_i)]+\text{c.c.}\right), \qquad (10)$$

where the k_i are equilateral n-angles of length $|k_i|=p_0$ and α_i are arbitrary phase angles.

The cases $n=1$ $(=2)$, 3, 4 have striped, triangular, and square order, respectively. Inserting (10) into the action, the effective lagrangians become

$$\mathscr{L}_n=\tfrac{1}{2}a\varphi_n^2-\tfrac{1}{3}c_n\varphi_n^3+\tfrac{1}{4}\mathrm{d}_n\varphi_n^4\ , \qquad (11)$$

with $c_1=0$, $c_3=2/\sqrt{6}$, $c=0$, $d_1=\frac{3}{2}$, $d_3=\frac{9}{2}$, $d_4=\frac{39}{8}$ where we have set, in the second case, $\alpha_1+\alpha_2+\alpha_3=2\pi\cdot$integer in order to get the largest possible cubic term which leads to the lowest energy. The lagrangians $\mathscr{L}_1$ and $\mathscr{L}_4$ have a second-order phase transition at $a=0$, i.e. at $\nu=\nu_c$. These transitions can, however, not take place since before this happens, at a precocious value $a_p=2c_3^2/9d_3=2^3/3^5$, the lagrangian $\mathscr{L}_3$ has a first-order transition into a state with triangular order, where φ_3 jumps from 0 to $3a_3/c_3=2^2\sqrt{6}/3^4$.

For $\nu \ll \nu_c$, however, the striped state has eventually the lower energy. Indeed, for whatever cubic term, a quartic term $\frac{1}{4}d\varphi^4$ leads to a ground state energy $\rightarrow -\frac{1}{8}a^2/d$ so that the energy with the smallest d is the lowest. Hence, at some $\nu<\nu_c$ there must be a further first-order transition triangular$\rightarrow$striped. This happens at $a\approx 0.24$.

It should be noted that if d is very large but not ∞, the striped state is destroyed immediately by fluctuations. In order to see this we allow for a space-dependent phase $\alpha(\xi)$ in the ansatz (10) and calculate from $\mathscr{L}_1$ the leading gradient terms $\propto \alpha(\partial_2^4-4p_0\partial_1^2)\alpha$ where ∂_2 acts parallel and ∂_1 orthogonal to the stripes. The divergence of $\int \mathrm{d}^2p/(p_1^4+4p_0p_2^2)$ is so violent (much more than in the 2D XY model) that the order of the striped state is destroyed. There remains, however, a directional memory of the stripes. In this respect, the striped state behaves like a two-dimensional smectic liquid crystal [16] which by fluctuations is always carried into the nematic order characterized by a director (an axial vector).

A similar procedure for the triangular case gives gradient energies for $\alpha_i(\xi)$ which describe the two-dimensional elasticity of the triangular pattern. Fluctuations will produce dislocations and disclinations which can carry the system into a liquid (and possible a hexatic) phase [17], i.e., a disordered ondulated pattern.

It thus appears as though a string with extrinsic curvature can have a rich variety of interesting phases which are certainly worth investigating further.

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