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ON THE ELECTROMAGNETIC COUPLINGS OF THE f AND σ MESONS

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Abstract: We use the Ward identities for the $\theta_{\mu\nu} V_\rho V_\lambda$ vertex and the assumptions of f and σ meson dominance of the stress energy momentum tensor $\theta_{\mu\nu}$, and of vector meson dominance of the electromagnetic currents V_ρ to present estimates for the electromagnetic couplings of the f and σ mesons. A comparison is made with results obtained by other methods.

1 INTRODUCTION

Recently, considerable effort has been devoted to investigations of the physical consequences of broken dilatation symmetry [1]. The underlying ideas of all methods used so far are abstracted from free or simple interacting field theories [2-4]. In particular, such motivated assumptions on the chiral dimensional structure of the Hamiltonian density [2, 3] and dimensional properties of the weak and electromagnetic currents J_μ [2] lead to Ward identities involving the hadron stress energy momentum tensor $\theta_{\mu\nu}$ and the currents J_μ. These can be used to study independently both the high and low energy behavior of various amplitudes, these regions being then related by dispersion relations. In this paper, we will be mainly interested in the applications of these Ward identities specifically to give information on the electromagnetic couplings of the f and σ mesons.

The situation concerning f meson dominance of the traceless part of the stress tensor has been discussed most thoroughly recently by Renner [5, 6]. The results obtained by making the simplest dispersive f dominance assumptions are in general in agreement with results obtained by Wess and Zumino [7] using an effective Lagrangian appropriately constructed so that

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the f meson couples universally to all hadrons "containing" no strange quarks ‡, in the form $\mathcal{L} \propto f_{\mu\nu}\,\theta^{\mu\nu}$. Remarkably this coupling may provide the basis for an explanation [5, 6, 8, 9] of the experimentally observed ‡‡ s channel helicity conservation at high energies in various diffractive processes. However, there is a disturbingly large discrepancy between this theory and that of Engels and Höhler's [10] present experimental estimate for the ratio $R^{(f)} \equiv (g^{(1)}_{fNN})/(g_{f\pi\pi})$; in fact $R^{(f)}_{exp}/R^{(f)}_{theor} \approx 3$. Here, at least two comments are relevant:

(1) the experimental value [10] for $g^{(1)}_{fNN}$ as determined from an analysis of πN backward scattering is probably subject to much larger uncertainties than the quoted 10% error ‡‡‡;

(2) $R^{(f)}_{theor}$ could be brought into agreement with present experiment by introducing subtractions for certain form factors without disturbing the s-channel helicity conservation result; however, this would rather spoil the simplicity of the present theoretical picture #.

One should perhaps reserve judgement until the value for $R^{(f)}_{exp}$ is better known. Renner [6] has also made maximal smoothness assumptions to obtain estimates for the radiative decays of the f meson: he obtains $\Gamma_{f\to\rho\gamma} \approx 1.3$ MeV; $\Gamma_{f\to\gamma\gamma} \approx 7$ KeV. The latter result on the two-photon coupling will hopefully be confronted with experimental data from electron electron scattering.

Although the σ meson is experimentally by no means well established, we will assume it to exist with a large mass and width $m_\sigma = 700$ MeV, $\Gamma_\sigma = 400$ MeV and to dominate the trace of the energy momentum tensor $\theta \equiv \theta^{\mu}_{\mu}$. In this case, standard assumptions on the nature of chiral and

‡ Here it is assumed, as it will be in the rest of this paper that the f' meson does not couple appreciably to hadrons "containing" no strange quarks.

‡‡ See refs. [1–4] of ref. [8].

‡‡‡ In fact, H. C. Schaile (Karlsruhe Thesis, 1970) using fixed u dispersion relations and R. Strauss (Karlsruhe Thesis, 1970) using fixed angle dispersion relations obtain similar estimates for $g^{(1)}_{fNN}$ but the error quoted are larger, $\sim 50\,\%$.

Crewther [11] has suggested that scale breaking effects might show up in a subtraction for the tensor form factor F_1 defined in the matrix element

$$\langle \pi(P+\tfrac{1}{2}q)\,|\,\theta_{\mu\nu}(0)\,|\,\pi(P-\tfrac{1}{2}q)\rangle = (2P_\mu P_\nu - J_{\mu\nu}(q)/6)\,F_1(q^2) + J_{\mu\nu}(q)\,F_2(q^2)\ ,$$

using collinear dispersion relations saturated with σ and π poles alone together with other assumptions (whose validities are just as questionable as assuming that F_1 is in fact of unsubtracted form) he obtains estimates

$$F_1'(0) \approx \frac{m^2_{A_1} + m^2_\sigma}{m^2_{A_1}\left(m^2_{A_1} - m^2_\sigma\right)} \approx \frac{9}{2m^2_f}\ , \qquad g_{\sigma\pi\pi}\,\gamma_\sigma^{-1} \approx \left(1 - \frac{m^2_\sigma}{m^2_{A_1}}\right).$$

Assuming then a once subtraced form for F_1 but the same unsubtractedness assumptions for the stress tensor form factors of the nucleon $R^{(\sigma)}_{exp}$ and $R^{(\sigma)}_{theor}$ would now agree well; however, $R^{(f)}_{exp}$ and $R^{(f)}_{theor}$ would now differ by a factor ≈ 10.

dilatational symmetry breaking force a subtraction ‡ in the pion matrix element $\langle \pi(p) | \theta(0) | \pi(p') \rangle$, although matrix elements $\langle T(p) | \theta(0) | T(p') \rangle_{(\text{spin average})}$ for other targets T can in principle be unsubtracted [12, 13]. Again there is a discrepancy between the experimental [10] estimate ‡‡ and the simplest theoretical estimate ‡‡‡ for $R^{(\sigma)} \equiv (g_{\sigma NN})/(g_{\sigma\pi\pi})$; here $R^{(\sigma)}_{\text{exp}}/R^{(\sigma)}_{\text{theor}} \approx 2$. Note, however, that since $R^{(\sigma)}_{\text{exp}}$ and $R^{(f)}_{\text{exp}}$ which we have taken come from the same analysis [10] of πN scattering the present discrepancies with $R^{(\sigma)}_{\text{theor}}$ and $R^{(f)}_{\text{theor}}$ are correlated †; one may, however, be too optimistic in hoping that after future analyses the experimental values for $R^{(\sigma)}$ and $R^{(f)}$ will be brought into simultaneous agreement with the corresponding theoretical values. On the other hand, making definite assumptions on the nature of chiral and dimensional breaking contained in the energy density, and absolute prediction for $g_{\sigma\pi\pi}$ is obtained [12, 13]; in particular the assumptions of $(3, 3^*) + (3^*, 3)$ breaking (with c given by either of the favourite values $c \simeq -1.25$ [15] or $c \simeq -0.25$ [16]) and only a c number δ [17] are consistent with a large $\Gamma_\sigma \approx 400$ MeV. The couplings $\sigma A\pi$ and σAA have also been considered [18] using the familiar hard meson methods [19], however, here the estimate for $g_{\sigma A\pi}$ depends critically on what one considers the natural smoothness assumption to make ††.

In this paper our primary aim is to augment the list of estimates for couplings obtained by making smoothness assumptions on the $\theta V_\rho V_\lambda$ vertex. In order to give a more coherent analysis in sect. 2, we also consider the $\theta_{\mu\nu} V_\rho V_\lambda$ vertex and discuss the associated f couplings. We show that a certain set of smoothness assumptions reproduce the vertex of Renner [6]. However, it is noteworthy that this vertex does not coincide with that obtained from the effective Lagrangian of Wess and Zumino [7] containing minimal f coupling. Nevertheless the two vertices reproduce the same onshell electromagnetic coupling constants $g_{fV\gamma}$ and $g_{f\gamma\gamma}$ and have the same properties with respect to s-channel helicity conservation [6].

‡ This subtraction should not a priori be thought of as being necessary to reproduce a specific high energy behaviour. They merely serve as an approximation to the tails of the higher resonances as seen in the low energy region.

‡‡ For another estimate for $R^{(\sigma)}_{\text{exp}}$ agreeing more with $R^{(\sigma)}_{\text{theor}}$, see ref. [14].

‡‡‡ See, however, footnote # on the previous page.

† As Renner [5] has remarked, a destructive interference between σ and f poles is needed in order to reproduce the structure of the data, and the fit may not be stable against addition of further contributions.

†† The simplest set of smoothness assumptions including the assumption of absence of a d wave σAA coupling (which corresponds to the structure of the simplest effective Lagrangian [7] lead to $g_{\sigma A\pi} \propto (d_S - 3)$, (when $d_S = 1$ the ratio $g_{\sigma A\pi}/g_{\sigma\pi\pi}$ is the same as that given by Gilman and Harari [20]). On the other hand, allowing possible non-smooth behaviour but requiring the F_+ form factor defined in $\langle \sigma | A_\mu | \pi \rangle$ to be unsubtracted one obtains (independently of d_S) approximately the Gilman-Harari value for $g_{\sigma A\pi}/g_{\sigma\pi\pi}$ provided that one assumes that $F_1(t)$ is at most once subtracted and $F_1'(0) \ll 1/m_\pi^2$ [18].

The results obtained for the σ couplings by making maximal smoothness assumptions are very simply stated. They are: (1) the s wave σVV coupling is proportional to $(d_S - 1)$ where d_S is the dimension of the spatial components of the electromagnetic current; (2) the d wave σVV coupling vanishes; (3) $g_{\sigma V\gamma} = 0$, and (4) $g_{\sigma\gamma\gamma} = 0$. We refer the reader to sect. 4 for a discussion of the consequences of the results and their comparison with results obtained by other methods.

2. WARD IDENTITIES AND f MESON COUPLINGS

The relevant object for our study is a covariantization $\tau^*_{\mu\nu\rho\lambda}$ of the three-point function

$$\tau^a_{\mu\nu\rho\lambda}(p,k) = \int e^{i(px+ky)} \langle 0 | T\theta_{\mu\nu}(0) V^a_\rho(x) V^a_\lambda(y) | 0 \rangle , \tag{1}$$

where $V^a_\rho(x)$ are the conserved vector currents $a = 3$ or 8. The most elegant way to obtain the Ward identities is by considering the variation of the transition amplitude with respect to variations of external electromagnetic and gravitational fields. This has been done by Wess and Zumino [7]; here we just state the results and refer the reader to ref. [7] for a full derivation.

First define the reduced vertex [for a particular SU(3) index a which we drop in the following],

$$\Gamma_{\mu\nu\rho\lambda}(p,k) \equiv \Delta^{-1}(p)\Delta^{-1}(k)\tau^*_{\mu\nu\rho\lambda}(p,k)\gamma_V^{-2} , \tag{2}$$

where $\Delta(p)$ is defined in terms of the conserved propagator

$$\Delta(p) J_{\rho\lambda}(p) \equiv \Delta^*_{\rho\lambda}(p) \equiv -i\int e^{-ipx} \langle 0 | T^* V_\rho(x) V_\lambda(0) | 0 \rangle . \tag{3}$$

Here $J_{\rho\lambda}$ denotes the spin one projection

$$J_{\rho\lambda}(p) = H_{\rho\lambda}(p,p) , \tag{4}$$

with

$$H_{\rho\lambda}(p,k) \equiv k_\rho p_\lambda - kpg_{\rho\lambda} .$$

The factor γ_V in (2) denotes the coupling strength of the dominant vector meson to V_μ ‡

$$\gamma_V^2 m_V^{-2} = \lim_{p^2 \to m_V^2} \frac{\partial}{\partial p^2} \Delta^{-1}(p) . \tag{5}$$

‡

$$\langle 0 | V_\mu(0) | V, p, \lambda, m_V \rangle \equiv \epsilon_\mu(p,\lambda) m_V^2/\gamma_V .$$

Then the properties of $\Gamma_{\mu\nu\rho\lambda}$ are the following ‡, [7]:

(A) $\Gamma_{\mu\nu\rho\lambda}$ has no pseudotensor components;

(B) Symmetry: $\Gamma_{\mu\nu\rho\lambda}(p,k) = \Gamma_{\nu\mu\rho\lambda}(p,k)$;

(C) Crossing symmetry: $\Gamma_{\mu\nu\rho\lambda}(p,k) = \Gamma_{\mu\nu\lambda\rho}(k,p)$;

(D) Gauge invariance: $p^\rho \Gamma_{\mu\nu\rho\lambda}(p,k) = 0 = \Gamma_{\mu\nu\rho\lambda}(p,k)k^\lambda$;

(E) Covariance: $\Gamma_{\mu\nu\rho\lambda}$ satisfies the Ward identity ($q = p + k$)

$$\gamma_V^2 q^\mu \Gamma_{\mu\nu\rho\lambda}(p,k) = -(g_{\lambda\nu} k^\mu - g^\mu_\lambda k_\nu) J_{\mu\rho}(p)\, \Delta^{-1}(k) - (g_{\rho\nu} p^\mu - g^\mu_\rho p_\nu) J_{\mu\lambda}(k)\, \Delta^{-1}(p) .$$

The properties (A) to (D) leave $\Gamma_{\mu\nu\rho\lambda}(p,k)$ expressible in terms of 13 form factors of definite crossing properties and the Ward identity (E) supplies six relations between these. To obtain estimates for the couplings of the f meson, it is necessary to invoke smoothness assumptions. Maximal smoothness assumptions directly on the vertex $\Gamma_{\mu\nu\rho\lambda}$ lead to Renner's relations [6] ($\Sigma \equiv p - k$, $\epsilon^\mu_\mu = 0$, $\epsilon^{\mu\nu} = \epsilon^{\nu\mu}$, $q^\mu \epsilon_{\mu\nu} = 0$, $\epsilon p = \epsilon' k = 0$):

$$\lim_{q^2 \to m_f^2} [(1 - q^2/m_f^2)\epsilon^{\mu\nu}(q)\, \epsilon^\rho(p)\, \epsilon'^\lambda(k)\, \Gamma_{\mu\nu\rho\lambda}(p,k)]$$

$$= -\tfrac{1}{2} \epsilon^{\mu\nu}(q)\, \epsilon^\rho(p)\, \epsilon'^\lambda(k) \{\Sigma_\mu \Sigma_\nu g_{\rho\lambda} - \Sigma_\mu g_{\nu\rho} p_\lambda - \Sigma_\nu g_{\mu\rho} p_\lambda + \Sigma_\mu g_{\nu\lambda} k_\rho + \Sigma_\nu g_{\mu\lambda} k_\rho - 2pk(g_{\mu\rho} g_{\nu\lambda} + g_{\nu\rho} g_{\mu\lambda})\} , \quad (6)$$

where no arbitrary parameters appear. The full analysis involves much tedious algebra and is outlined in the appendix.

The relation (6) leads to a fVV coupling given by the effective Lagrangian $\mathcal{L}_{fVV} = -\gamma_f m_f^{-1} f_{\mu\nu} V^{\mu\rho} V^\nu_\rho$ (where $V_{\mu\rho} = \partial_\mu V_\rho - \partial_\rho V_\mu$) which differs from the universal coupling ‡‡, [7]

$$\mathcal{L}_{fVV} = \gamma_f m_f^{-1} f_{\mu\nu} \theta_V^{\mu\nu} = -\gamma_f m_f^{-1} f_{\mu\nu} (V^{\mu\rho} V^\nu_\rho - m_V^2 V^\mu V^\nu) . \quad (7)$$

The different couplings, however, have the same property of conserving s-channel helicity [6]. In addition both relation (6) and the Lagrangian of Wess and Zumino [7] yield the same estimates for the radiative decay widths

$$\Gamma_{f \to V\gamma} \approx (\gamma_f/\gamma_V)^2\, 0.29 \text{ MeV} , \quad (8)$$

‡ There should be no anomalies [21] in eqs. (D) and (E) since the invariance form which they follow must be exact.

‡‡ One can reproduce Renner's vertex within the effective Lagrangian approach [7], however, this would involve the addition of non-minimal interaction terms for the f meson.

and ‡

$$\Gamma_{f\to\gamma\gamma} \approx \frac{\gamma_f^2}{\gamma_\rho^4}\left(1+\frac{\gamma_\rho^2}{\gamma_\omega^2}\right)^2 43\ \text{keV}\ , \tag{9}$$

where γ_f is defined in the coupling of the f meson to the energy momentum tensor

$$\langle 0\,|\,\theta_{\mu\nu}(0)\,|\,f,q,\lambda\rangle = \epsilon_{\mu\nu}(q,\lambda)\,\gamma_f^{-1}\,m_f^3\ . \tag{10}$$

With γ_f estimated from the f width and the universal coupling

$$\mathcal{L}_{f\pi\pi} = \gamma_f\, m_f^{-1} f_{\mu\nu}\,\theta_\pi^{\mu\nu} = \gamma_f\, m_f^{-1} f_{\mu\nu}\,\partial^\mu\pi\partial^\nu\pi\ , \tag{11}$$

one obtains ‡‡, [5]

$$\gamma_f^2/4\pi = 10.8\ . \tag{12}$$

Then using $(\gamma_\rho^2/4\pi) = 2.5$ and $(\gamma_\omega^2/4\pi) = 22.5$ one obtains the following estimates for the widths $\Gamma_{f\to\rho\gamma} = 1.3$ MeV and $\Gamma_{f\to\gamma\gamma} = 7$ keV. The latter result agrees well with an estimate of $\Gamma_{f\to\gamma\gamma} = 5.7$ MeV from finite energy sum rules [22]; with this coupling the f can be produced in e^-e^- collisions with a cross section of approximately 5.10^{-34} cm^2 at $\sqrt{s} = 6$ GeV. Hopefully these estimates will soon be confronted with experiment.

‡ We have included only ω, ρ contributions and have approximated $m_\rho \approx m_\omega$;

$$\mathcal{L}_{fV\gamma} = e g_{fV\gamma}\, m_f^{-1} f_{\mu\nu}\, V^{\mu\rho} F_\rho^\nu$$

gives

$$\Gamma_{f\to V\gamma} = \tfrac{1}{40}\alpha g_{fV\gamma}^2 m_f \left(1-\frac{m_V^2}{m_f^2}\right)^3 \left(1+\frac{1}{2}\frac{m_V^2}{m_f^2}+\frac{1}{6}\frac{m_V^4}{m_f^4}\right),$$

and

$$\mathcal{L}_{f\gamma\gamma} = \tfrac{1}{2} e^2 g_{f\gamma\gamma}\, m_f^{-1} f_{\mu\nu} F^{\mu\rho} F_\rho^\nu$$

gives

$$\Gamma_{f\to\gamma\gamma} = \tfrac{1}{20}\,(\alpha^2\pi g_{f\gamma\gamma}^2 m_f)\ .$$

‡‡

$$\Gamma_{f\to\pi\pi} = \frac{\gamma_f^2}{4\pi}\frac{2}{5}\frac{k_\pi^5}{m_f^4} = 150\ \text{MeV}\ .$$

3. TRACE IDENTITY AND MAXIMAL SMOOTHNESS ASSUMPTIONS

The relevant object for a study of the electromagnetic couplings of the σ meson is the conserved covariantization $\tau^*_{\rho\lambda}(p, k)$ of the three-point function

$$\tau_{\rho\lambda}(p, k) = \int e^{i(px+ky)} \langle 0 | T\theta(0) V_\rho(x) V_\lambda(y) | 0 \rangle . \tag{13}$$

It has been shown [7] that $\tau^*_{\rho\lambda}(p, k)$ is indeed simply related to the conserved covariantization $\tau^*_{\mu\nu\rho\lambda}(p, k)$ by the trace identity‡:

$$\tau^*_{\rho\lambda}(p, k) = \tau^{*\mu}_{\mu\rho\lambda}(p, k) . \tag{14}$$

One can now deduce, for the reduced vertex

$$\Gamma_{\rho\lambda}(p, k) \equiv \Delta^{-1}(p) \Delta^{-1}(k) \tau^*_{\rho\lambda}(p, k) \gamma_V^{-2} , \tag{15}$$

the low energy theorem

$$\gamma_V^2 \Gamma_{\rho\lambda}(p, -p) = 2 J_{\rho\lambda}(p) p^2 \frac{\partial}{\partial p^2} \Delta^{-1}(p) \tag{16}$$

by using the Ward identity (E) and the trace identity (15).

Now expressing $\Gamma_{\rho\lambda}(p, k)$ in terms of its two independent form factors‡‡:

$$\Gamma_{\rho\lambda}(p, k) \equiv W_1 H_{\rho\lambda}(p, k) + \frac{W_2}{m_V^2} J_{\rho\mu}(p) J^\mu_\lambda(k) , \tag{17}$$

we have

$$W_1(0, p^2, p^2) - \frac{p^2}{m_V^2} W_2(0, p^2, p^2) = 2\gamma_V^{-2} p^2 \frac{\partial}{\partial p^2} \Delta^{-1}(p) , \tag{18}$$

which gives the rigorous result

$$W_1(0, 0, 0) = 0 . \tag{19}$$

To obtain estimates for the σ couplings one now has to make smoothness assumptions.

(1) The most naive smoothness assumption would now be to take $W_i(0, p^2, k^2)$ as constants for small p^2, k^2. This would only be consistent in so far as $\Delta(p)$ is vector meson pole dominated

$$\Delta^{-1}(p) = \frac{\gamma_V^2}{m_V^2} (p^2 - m_V^2) . \tag{20}$$

Then making a σ pole dominance assumption one would obtain

‡ Eq. (14) could have anomalies [2], however, we ignore this possibility.

‡‡ Note that $W_i = W_i(q^2, p^2, k^2) = W_i(q^2, k^2, p^2)$.

$$W_1 = 0 ,$$

$$W_2 = 2m_\sigma^2/(q^2 - m_\sigma^2) . \tag{21}$$

(2) On the other hand, one may prefer to recover the results of effective Lagrangian models incorporating vector meson dominance of the electromagnetic currents and σ dominance of θ and where the dimension of the spatial component of the electromagnetic current has a fixed dimension d_s ‡

$$i[\int d^3x x^\mu \theta_{\mu 0}(x),\ V_i(y)]_{x_0=y_0} = (y\partial + d_s) V_i(y) . \tag{22}$$

In this case smoothness assumptions must be made not for the gauge invariant covariantization $\tau^*_{\rho\lambda}$ but rather for the covariantization $\tau^{**}_{\rho\lambda}$ which has the same spatial components as $\tau_{\rho\lambda}$ defined in (13). The two covariantization differ by sea-gulls containing only single poles ‡‡.

$$\tau^{**}_{\rho\lambda}(p,k) = \tau^{*}_{\rho\lambda}(p,k) - g_{\rho\lambda}\Delta_{\theta S}(q) + (d_s - 3)[\Delta^{*}_{\rho\lambda}(p) + \Delta^{*}_{\rho\lambda}(k) - S g_{\rho\lambda}] \tag{23}$$

where $S(y)$ is the operator appearing in the commutator ‡‡‡ ;

$$i[V_0(x),\ V_\lambda(y)]_{x_0=y_0} = S(y) g_{i\lambda}\partial^i_x \delta^3(\boldsymbol{x}-\boldsymbol{y}) , \tag{24}$$

$$\Delta_{\theta S}(q) \equiv -i \int e^{-iqx} \langle 0 | T\theta(x) S(0) | 0 \rangle , \tag{25}$$

$$S = \langle 0 | S(0) | 0 \rangle . \tag{26}$$

The covariantization $\tau^{**}_{\rho\lambda}(p,k)$ satisfies the Ward identity

$$p^\rho \tau^{**}_{\rho\lambda}(p,k) = -p_\lambda \Delta_{\theta S}(q) + (d_s - 3) p^\rho \Delta^{**}_{\rho\lambda}(k) , \tag{27}$$

and the low energy theorem †

$$\tau^{**}_{\rho\lambda}(p,-p) = (2d_s - 4 - p\frac{\partial}{\partial p}) \Delta^{**}_{\rho\lambda}(p) , \tag{28}$$

where $\Delta^{**}_{\rho\lambda}(p)$ is the non-conserved propagator

‡ One can also prove [23]

$$i[\theta(x),\ V_0(y)]_{x_0=y_0} = (3 - d_s) V_i(x)\partial^i_x \delta^3(\boldsymbol{x}-\boldsymbol{y})$$

+ higher order S. T. vanishing on integration over $\boldsymbol{x}$.

‡‡ For a derivation of this relation, see ref. [18].

‡‡‡ In cases where the equal time commutator is divergent we consider the derivation as formal only.

† Since we expect the high energy behaviour of the propagator to be determined by d_s, we expect that one can make arguments for the neglection of $\tau^{**}_{\rho\lambda}(p,-p)$ in (28) for high spacelike p^2.

$$\Delta^{**}_{\rho\lambda}(p) = \Delta^{*}_{\rho\lambda}(p) - Sg_{\rho\lambda} \,. \tag{29}$$

In the following we consider dominance of matrix elements of V^3 and V^8 by one vector meson in each case although a similar analysis can be carried out for the case of two resonances. We first define the vertex $\Gamma^{**}_{\chi\tau}(p, k)$ by

$$\tau^{**}_{\rho\lambda}(p, k) = \Delta^{**}_{\rho\chi}(p)\,\Delta^{**}_{\lambda\tau}(k)\,\Gamma^{**}_{\chi\tau}(p, k)\,\frac{1}{q^2 - m_\sigma^2}\,\frac{\gamma_V^2}{m_V^2} \,. \tag{30}$$

Then assuming in the low energy region ‡

(i) $$\Delta_{\theta S}(q) \approx -\frac{\Delta_{\theta S}(0)\, m_\sigma^2}{q^2 - m_\sigma^2}$$

(ii) $$\Delta^{**}_{\rho\lambda}(p) = \frac{m_V^2}{\gamma_V^2}\,\frac{p_\rho p_\lambda - m_V^2 g_{\rho\lambda}}{p^2 - m_V^2}$$

(iii) that $\Gamma^{**}_{\chi\tau}(p, k)$ is well approximated by a tensor containing only up to quadratic terms in the momenta, we find the result, expressed in terms of W_1, W_2

$$W_1 = 0 \,,$$

$$W_2 = \{2m_\sigma^2 - q^2(3 - d_S)\}/(q^2 - m_\sigma^2) \,. \tag{31}$$

Eq. (31) implies that $\Gamma_{\rho\lambda}(p, k)$ is smoothest when $d_S = 3$ and in this case the result agrees with (21). The result (31) is that obtained first by Wess and Zumino [7] in the framework of effective Lagrangians.

4. RESULTS AND CONSEQUENCES FOR σ COUPLINGS

In this section we discuss the consequences of the results (19), (21) and (31), and make a comparison with results obtained by other methods.

(A) Independently of any assumption on $\Delta(p)$, the result (19) implies that in any Lagrangian model satisfying the trace identity [14] and in which θ is a good interpolating field for σ, the contribution of the σ exchange Feynman diagram to the invariant amplitudes of Compton scattering on any target vanishes in the forward direction.

(B) σVV couplings: the on-shell coupling constants defined by the effective Lagrangian

$$\mathcal{L}_{\sigma VV} = \tfrac{1}{2} g_{\sigma VV}\, m_V\, \sigma V^\mu\, V_\mu + h_{\sigma VV}\,\frac{1}{m_V}\,\sigma \partial_\mu V_\nu\, \partial^\nu V_\mu \tag{32}$$

are given by

‡ Note $\Delta_{\theta S}(0) = (d_S - 1)S$.

$$h_{\sigma VV} = 0 \tag{33}$$

for both cases (21) or (31); and by

$$g_{\sigma VV} = (1 - d_S) \frac{m_V}{m_\sigma} \gamma_\sigma \tag{34a}$$

or

$$g_{\sigma VV} = -2 \frac{m_V}{m_\sigma} \gamma_\sigma \tag{34b}$$

for (21) and (31) respectively where γ_σ defines the strength of the σ meson to $\theta_{\mu\nu}$,

$$\langle 0 \left| \theta_{\mu\nu}(0) \right| \sigma(q) \rangle = -\tfrac{1}{3} \gamma_\sigma^{-1} m_\sigma J_{\mu\nu}(q) \,. \tag{35}$$

We recall that by making the standard assumption that the chiral breaker in θ_{00} is a Lorentz scalar [15] of dimension d‡ and assuming also maximal smoothness of the $\theta \partial A \partial A$ vertex one obtains#, [12, 13]

$$g_{\sigma\pi\pi} = \gamma_\sigma \left[1 + (d - 2) \frac{m_\pi^2}{m_\sigma^2} \right] . \tag{36}$$

Note that the results (34a) and (34b) agree for $d_S = 3$. There is in fact now quite an accumulation of indirect evidence supporting the quark model value $d_S = 3$, these being (i) the phenomenon of scaling in deep inelastic lepton scattering and its explanation using operator product expansions on the light cone [24]; (ii) accepting naive arguments [25] the total electron positron annihilation cross-section should go as $s^{1/2(d_S-5)}$; the preliminary data [26] favour $d_S = 3$ (although it is by no means clear that the asymptotic region has been reached); (iii) the longitudinal to transverse ratio in deep inelastic electron proton scattering favours quark model commutators [27]; (iv) present data is consistent with these being only a c-number δ in θ_{00} [17] - $d_S \neq 3$ would require the presence of a q-number δ.

(C) $\sigma V\gamma$ coupling: defining the on-shell $\sigma V\gamma$ coupling constant $g_{\sigma V\gamma}$ by

‡ Hopefully d can eventually be measured in deep inelastic lepton scattering. See, e.g., Fritzsch and Gell-Mann [1].

$$\mathcal{L}_{\sigma\pi\pi} = \tfrac{1}{2} m_\sigma g_{\sigma\pi\pi} \sigma\pi^2 : \Gamma_{\sigma \to \pi\pi} = \frac{3}{4} \frac{g_{\sigma\pi\pi}^2}{4\pi} k_\pi = 400 \text{ MeV}$$

with

$$m_\sigma = 400 \text{ MeV}$$

gives

$$\gamma_\sigma \approx g_{\sigma\pi\pi} \approx \frac{m_\sigma}{\sqrt{2} f_\pi} \approx \gamma_\rho \,.$$

$$\mathcal{L}_{\sigma V\gamma} = e g_{\sigma V\gamma} m_\sigma^{-1} \sigma F_{\mu\nu} V^{\mu\nu} , \tag{37}$$

we see from (21) and/or (31) that maximal smoothness implies

$$g_{\sigma V\gamma} = 0 , \tag{38}$$

as was first observed by Ellis [12] using a Lagrangian model. This would then imply that σ production in e^+e^- collisions is not enhanced by going to a pole $s \approx m_V^2$ as was hoped for by Creutz and Einhorn ‡ [28].

(D) $\sigma\gamma\gamma$ coupling: defining the on-shell $\sigma\gamma\gamma$ coupling constant by the effective Lagrangian ⌗

$$\mathcal{L}_{\sigma\gamma\gamma} = e^2 g_{\sigma\gamma\gamma} \tfrac{1}{2} m_\sigma^{-1} \sigma F_{\mu\nu} F^{\mu\nu} , \tag{39}$$

one obtains, again from the smoothness results (21) and/or (31)

$$g_{\sigma\gamma\gamma} = 0 . \tag{40}$$

Note that both the results (39) and (40) differ considerably from those obtained by a naive application of vector meson dominance in the frame of the decaying σ to (34) ⌗⌗ [unless $d_S = 1$ in case (34a)].

The result (40) also differs from the estimate of $g_{\sigma\gamma\gamma}$ obtained by Schrempp, Schrempp and Walsh [22] using finite energy sum rules for pion Compton scattering, which is equivalent to

$$g^2_{\sigma\pi\pi} g^2_{\sigma\gamma\gamma} \approx 4 . \tag{41}$$

The estimate (41) has also been elsewhere in the literature, albeit on the basis of quite dubious assumptions †.

A non-vanishing coupling constant $g_{\sigma\gamma\gamma}$ can in principle be obtained in the Ward identity approach. To do this we have to admit, however, non-maximal smoothness behaviour for the $\Gamma^{**}_{\mu\nu}$ vertex. The simplest thing that can be done perhaps is to make the same pole dominance assumptions as before but now allow general fourth order momentum dependence in $\Gamma^{**}_{\mu\nu}$. Then one still has W_2 as given in (31) but an arbitrary parameter appears in W_1

‡ Creutz and Einhorn in fact proposed going to the ϕ pole (so that the γ would be emitted with sufficiently high momentum); however, as these authors noted in their original paper one might have an independent suppression in this case if σ does not couple to particle containing no non-strange quarks.

⌗
$$\Gamma_{\sigma\gamma\gamma} = \frac{e^4 m_\sigma}{4} \frac{g^2_{\sigma\gamma\gamma}}{4\pi} .$$

⌗⌗ One would in fact obtain $g_{\sigma\gamma\gamma} \approx (1/\gamma_\rho^2)\, \gamma_\sigma (1 - d_S)$. This agrees well with (41) for $d_S = 3$, however, this agreement is probably coincidental and we cannot easily justify the naive vector meson dominance assumption within our Ward identity approach.

† Footnote: see next page.

$$W_1 = \frac{\xi q^2}{q^2 - m_\sigma^2} , \tag{42}$$

allowing a non-vanishing $g_{\sigma\gamma V}$ and $g_{\sigma\gamma\gamma}$:

$$g_{\sigma\gamma V} = \tfrac{1}{2} \frac{\gamma_\sigma}{\gamma_V} \xi_V , \tag{43}$$

and including just ω, ρ contributions:

$$g_{\sigma\gamma\gamma} \approx \tfrac{1}{2} \gamma_\sigma \left(\frac{1}{\gamma_\rho^2} \xi_\rho + \frac{1}{\gamma_\omega^2} \xi_\omega \right) . \tag{44}$$

If we make in addition a quark model type of assumption $\xi_V = \xi$ for all V, we obtain the rough estimates (neglecting mixing effects and γ_ω^{-2} with respect to γ_ρ^{-2})

$$g_{\sigma\gamma V} \approx \frac{\gamma_\rho^2}{\gamma_V} g_{\sigma\gamma\gamma} . \tag{45}$$

Then the Schrempp et al. estimate (41) for $g_{\sigma\gamma\gamma}$ would give, together with (45) and using $g_{\sigma\pi\pi} \approx \gamma_\rho$

$$g_{\sigma\gamma V} \approx -2\, g_{\sigma\pi\pi}/\gamma_V . \tag{46}$$

This gives the same order of magnitude for the coupling constant $g_{\sigma\phi\gamma}$ as assumed by Creutz and Einhorn [28].

The simplest way to distinguish between the two results (40) and (41) is to observe the cross section of Compton scattering on pions when the two pions are in the neighbourhood of the σ mass. According to the maximal result (40) σ should not be produced at all. On the other hand the large estimate for the $\sigma\gamma\gamma$ coupling (41) should suffice to lift the σ out of the background.

At present there are two feasible experiments which can measure $\sigma_{\gamma\gamma \to \pi\pi}$:

† There has been an attempt by Sarker [29] to estimate $g_{\sigma\pi\pi}\, g_{\sigma\gamma\gamma}$ by using a superconvergent sum rule for the helicity flip amplitude $M_{1,-1}$ for pion Compton scattering. Apart from a mistake in the A_2 graph (which should read

$$M_{1,-1} = \frac{-e^2 g_{A_2\pi\gamma}^2\, t}{32} \left\{ \frac{3(s-u)^2 + 4st}{s - m_{A_2}^2} + s \leftrightarrow u \right\} ,$$

and which in turn drastically changes his bound $g_{\sigma\pi\pi}\, g_{\sigma\gamma\gamma} \lesssim -1 \pm 0.3$ obtained by saturating the sum rule with σ and higher resonances to $g_{\sigma\pi\pi}\, g_{\sigma\gamma\gamma} \lesssim -2.3 \pm ?$) we do not consider the assumption of superconvergence of $M_{1,-1}$ justified. Brodsky et al. [30] point out two similar ways of reproducing the estimate (41) both equally unconvincing. The first is to saturate Sarker's sum rule with the σ alone; and the second is to require that the forward differential cross section for $\gamma\gamma \to \pi^+\pi^-$ falls faster at high energy than either the pion Born term contribution or the σ contribution alone.

(1) collisions of e^-e^- at small angles producing two pions ‡;

(2) photoproduction of two pions on a high Z nucleus ar small momentum transfer.

5. CONLCUSION

The theoretical situation concerning the coupling constants $g_{\sigma V\gamma}$ and $g_{\sigma\gamma\gamma}$ is by no means clear. The most natural assumption of maximal smoothness within the Ward identity approach leads to a vanishing of these coupling constants - in which case σ will be absent in most electromagnetic reactions where one could hope best to see it. It remains for experiment to decide how good the maximal smoothness assumption for the $\theta V_\rho V_\lambda$ vertex really is.

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APPENDIX

The vertex $\Gamma_{\mu\nu\lambda\rho}$

The vertex $\Gamma_{\mu\nu\lambda\rho}(p,k)$ satisfying properties (A) to (D) is given in terms of 13 form factors of definite crossing properties which we first write in such a form that each form factor multiplies a gauge invariant quantity

$$\begin{aligned}\Gamma_{\mu\nu\rho\lambda}(p,k) = H_{\rho\lambda}(p,k)[&\theta_1\, g_{\mu\nu} + \frac{\theta_2}{m_V^2} p_\mu p_\nu + \frac{\tilde\theta_2}{m_V^2} k_\mu k_\nu] \\ &+ J_{\rho\tau}(p) J_\lambda^\tau(k) g_{\mu\nu}\, \theta_3/m_V^2 \\ &+ \tfrac{1}{2}[\{ \frac{\theta_4}{m_V^4} p_\nu J_{\rho\mu}(p) k^\tau H_{\tau\lambda}(p,k) + \frac{\tilde\theta_4}{m_V^2} k_\nu J_{\mu\lambda}(k) p^\tau H_{\rho\tau}(p,k) \\ &+ \frac{\theta_5}{m_V^2} J_{\rho\mu}(p) H_{\nu\lambda}(p,k) + \frac{\tilde\theta_5}{m_V^2} J_{\mu\lambda}(k) H_{\rho\nu}(p,k) \\ &+ \frac{\theta_6}{m_V^2} H_{\rho\mu}(p,k) H_{\nu\lambda}(p,k) + \frac{\theta_7}{m_V^2} J_{\rho\mu}(p) J_{\nu\lambda}(k) \\ &+ \frac{\theta_8}{m_V^2} p_\nu (p_\mu g_{\rho\tau} - p_\tau g_{\rho\mu}) J_\lambda^\tau(k) + \frac{\tilde\theta_8}{m_V^2} k_\nu (k_\mu g_{\lambda\tau} - k_\tau g_{\lambda\mu}) J_\rho^\tau(p) \\ &+ \theta_9 (g_{\rho\lambda} k_\mu p_\nu - g_{\nu\rho} k_\mu p_\lambda - g_{\mu\lambda} p_\nu k_\rho + kp g_{\mu\lambda} g_{\nu\rho})\} \\ &+ \{\mu \leftrightarrow \nu\}] \,, \end{aligned} \tag{A.1}$$

‡ A width $\Gamma_{\sigma\gamma\gamma} = 20$ keV leads to a cross section $\sigma_{ee\to ee\sigma} = 2.7 \times 10^{-33}$ cm^2 at $\sqrt{s} = 6$ GeV.

where

$$\theta_i \equiv \theta_i(q^2, p^2, k^2) , \qquad \tilde{\theta}_i(q^2, p^2, k^2) \equiv \theta_i(q^2, k^2, p^2) ,$$

$$\theta_i = \tilde{\theta}_i \quad \text{for} \quad i = 1, 3, 6, 7, 9 . \tag{A.2}$$

Define

$$\theta_i^{\pm} = \tfrac{1}{2}(\theta_i \pm \tilde{\theta}_i) . \tag{A.3}$$

The Ward identity can now ne shown to give six independent relations among the $\theta_i^{\pm}$.

Now consider the quantity $\Gamma_{\mu\nu\rho\lambda}\epsilon^{\rho}(p)\epsilon'^{\lambda}(k)$ and decompose it in a form such that only six form factors (G_i, $i = 1, \ldots, 6$)‡ remain non-zero when $p^2 = k^2 = m_V^2$

$$\Gamma_{\mu\nu\rho\lambda}\epsilon^{\rho}\epsilon'^{\lambda} = G_1\epsilon\epsilon'\Sigma_\mu\Sigma_\nu - \frac{1}{m_V^2}G_2\epsilon k\epsilon' p\Sigma_\mu\Sigma_\nu$$

$$- G_3(\epsilon k[\epsilon'_\mu\Sigma_\nu + \epsilon'_\nu\Sigma_\mu] - \epsilon' p[\epsilon_\mu\Sigma_\nu + \epsilon_\nu\Sigma_\mu])$$

$$+ G_4(\epsilon k[\epsilon'_\mu q_\nu + \epsilon'_\nu q_\mu] + \epsilon' p[\epsilon_\mu q_\nu + \epsilon_\nu q_\mu] - 2\epsilon k\epsilon' p g_{\mu\nu} - q^2[\epsilon_\mu\epsilon'_\nu + \epsilon_\nu\epsilon'_\mu])$$

$$+ G_5\epsilon\epsilon' J_{\mu\nu}(q) - \frac{1}{m_V^2}G_6\epsilon k\epsilon' pJ_{\mu\nu}(q)$$

$$+ G_7(\epsilon k[\epsilon'_\mu\Sigma_\nu + \epsilon'_\nu\Sigma_\mu] + \epsilon' p[\epsilon_\mu\Sigma_\nu + \epsilon_\nu\Sigma_\mu])$$

$$+ G_8(\epsilon k[\epsilon'_\mu q_\nu + \epsilon'_\nu q_\mu] - \epsilon' p[\epsilon_\mu q_\nu + \epsilon_\nu q_\mu])$$

$$+ G_9\epsilon k\epsilon' p g_{\mu\nu} + G_{10}m_V^2(\epsilon_\mu\epsilon'_\nu + \epsilon_\nu\epsilon'_\mu) + G_{11}\epsilon\epsilon' g_{\mu\nu}m_V^2 , \tag{A.4}$$

only G_5 and G_6 have σ poles. The G_i can be of course expressed in terms of the $\theta_i^{\pm}$. A particularly important result of the Ward identity (E) is that one has at $q^2 = 0$

$$-G_1(0, p^2, p^2) = G_3(0, p^2, p^2) = \tfrac{1}{2}\gamma_V^{-2}\left\{p^2\frac{\partial}{\partial p^2} - 1\right\}\Delta^{-1}(p) , \tag{A.5}$$

giving in particular

$$-G_1(0, m_V^2, m_V^2) = G_3(0, m_V^2, m_V^2) = \tfrac{1}{2} , \tag{A.6}$$

as noted by Renner [6].

Now follow the smoothness assumptions required to produce Renner's result.

Smoothness ansatz (I):

‡These correspond to the form factors defined by Renner [6].

Suppose

$$\lim_{q^2 \to m_f^2} \left(1 - \frac{q^2}{m_f^2}\right) \Gamma_{\mu\nu\rho\lambda}$$

is well approximated by tensors involving only fourth order in the momenta p, k. Then defining the residues

$$R_i^{\pm}(p^2, k^2) = \lim_{q^2 \to m_f^2} \left(1 - \frac{q^2}{m_f^2}\right) \theta_i^{\pm}(q^2, p^2, k^2) , \tag{A.7}$$

$$r_i(p^2, k^2) = \lim_{q^2 \to m_f^2} \left(1 - \frac{q^2}{m_f^2}\right) G_i(q^2, p^2, k^2) , \tag{A.8}$$

one has

$$R_2^- = R_4^- = R_5^- = R_8^- = 0 = R_4^+ ;$$

$$R_2^+, R_3, R_5^+, R_6, R_7, R_8^+ ,$$

are constants and R_1, R_9 are linear in $(p^2 + k^2)$.

Using the Ward identities it can now be shown that the r_i can be expressed in terms of three free parameters

$$r_1 = r_5 = -\tfrac{1}{4} \frac{m_f^2}{m_V^2} R_8^+ ,$$

$$r_2 = \tfrac{1}{4} (R_5^+ - R_8^+) ,$$

$$r_3 = \tfrac{1}{8} \frac{m_f^2}{m_V^2} (R_5^+ + R_8^+) ,$$

$$r_4 = -\tfrac{1}{8} \frac{m_f^2}{m_V^2} (R_5^+ + R_8^+) + \frac{1}{4m_V^2} (p^2 + k^2) R_8^+ ,$$

$$r_6 = -\tfrac{1}{4} (2R_6 - R_5^+ + R_8^+) ,$$

$$r_7 = \frac{1}{8m_V^2} (p^2 - k^2)(R_5^+ - R_8^+) ,$$

$$r_8 = \frac{1}{8m_V^2} (p^2 - k^2)(R_5^+ + R_8^+) ,$$

$$r_9 = r_{13} = 0 ,$$

$$r_{10} = -\frac{1}{8m_V^4}(p^2 - k^2)^2 (R_5^+ - R_8^+) ,$$

$$r_{11} = -\frac{1}{4m_V^4}(p^2 - k^2)^2 R_8^+ , \qquad r_{12} = \frac{1}{4m_V^2}(p^2 - k^2) R_8^+ . \tag{A.9}$$

Now defining

$$G_i(q^2, p^2, k^2) = r_i(q^2, p^2, k^2)\left(1 - \frac{q^2}{m_f^2}\right)^{-1} , \tag{A.10}$$

we now make the *second unsubtractedness ansatz* (II):

$$r_i(q^2, p^2, k^2)$$

is independent of q^2 for $i = 1, 3$. Then we have

$$-r_1 = r_3 = \tfrac{1}{2} , \tag{A.11}$$

and hence R_8^+, R_5^+ are determined from (A.9)

$$R_8^+ = \frac{2m_V^2}{m_f^2} = R_5^+ . \tag{A.12}$$

It now follows that all the residues at $q^2 = m_f^2$ of all the six form factors (G_i, $i = 1, 2, 3, 4, 7, 10$) which remain on contraction with the polarization tensor $\epsilon_{\mu\nu}(q)$ are well determined and we obtain Renner's [6] result (6).

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