Critical behavior of Ginzburg-Landau model coupled to massless Dirac fermions

Hagen Kleinert* and Flavio S. Nogueira†

Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, D-14195 Berlin, Germany

(Dated: Received May 10, 2002)

We point out interesting effects of additional massless Dirac fermions with $N_{\rm F}$ colors upon the critical behavior of the Ginzburg-Landau model. For increasing $N_{\rm F}$, the model is driven into the type II regime of superconductivity. The critical exponents are given as a function of $N_{\rm F}$.

tI ook ou last sen tencein abstract since the GI model has nev photon energy

PACS numbers: 74.20.-z, 05.10Cc, 11.25.Hf

I. INTRODUCTION

The critical fluctuations in the Ginzburg-Landau (GL) model of superconductors are an old problem in condensed matter physics[1]. While the underlying complex order field theory with $|\phi|^4$-interaction is well understood,[2] no satisfactory approximation has been found for a long time to deal with the additional gauge field. This may seem surprising since the Lagrangian is quadratic on the gauge field $\mathbf{A}$. One has therefore expected that $\mathbf{A}$ can be integrated out in a reasonable approximation to obtain an effective action with extra terms in the order field ϕ.[1] This is exactly possible for constant $|\phi|$ where a mean-field approximation for the effective potential receives an extra term $\sim -|\phi|^d$ in d dimensions. In four dimensions, where the model is relevant to particle physics, the extra term is $\sim |\phi|^4 \ln|\phi|^2$.[3] Such an extra term, if present in the full effective potential, would make the second-order phase transition first-order. As similar conclusion is derived from a one-loop renormalization group (RG) calculation in $d = 4-\epsilon$ dimensions, which shows no non-trivial charged fixed point[1,4], even up to two loops[5,6]. If the GL model is generalized in a such a way as to contain $N/2$ complex scalars instead of one, then non-trivial charged fixed points are found at one loop for $N > N_c = 365.9$. This geological number is considerably decreased at two loops[6] to $N_c = 36$, but it remains unphysically large. Thus there is no RG explanation for the experimetally observed second-order phase transition at $N = 2$.

A more significant reduction of the critical value of N is achieved by a RG approach in a fixed dimension $d \in (2,4)$. As we shall see in Section III, a one-loop calculation in $d = 3$ reduces N_c less than a third of the above value. This leads us to expect that non-trivial charged fixed points are more accessible in $d = 3$ than in $4-\epsilon$ dimensions. Indeed, this was recently confirmed by the present authors[7] by finding such a fixed point at $N = 2$ in a new three-dimensional RG calculation *below* T_c. The success of this approach relies on the explicit presence of two mass scales in the problem, defined by the inverse of the correlation length ξ and penetration depth λ. This is in contrast to all previous studies which were done in the disordered phase at $T \geq T_c$ which has only one physical mass scale. It has been proposed to introduce a second scale by assuming different renormalization points for each coupling of the theory[8]. Such a procedure has in fact led to a charged fixed points at $N = 2$, $d = 3$ provided the ratio between the two scales is sufficiently large[8]. Unpleasantly though, this ratio must be fixed by conditions external to the formalism, and a duality result[11] for a tricritical point was used to do so. Thus, in contrast to the recent finding for $T \leq T_c$, the RG-theoretic explanation of a charged fixed point for $T \geq T_c$ at $N = 2$ has remained obscure.

The purpose of this paper is to point out the interesting modifications of the critical behavior of the GL model with $N/2$ complex field ϕ brought about by the presence of extra massless Dirac fermion fields ψ with $N_{\rm F}$ replica. The introduction of massles Dirac fermions is of great interest, since effective microscopic models of strongly correlated electrons usualy contain them[9,10].

The bare Lagrangian is assumed to be

$$\mathcal{L} = \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GL}, \tag{1}$$

$$\mathcal{L}_{\rm F} = \bar{\psi}_0 \gamma_\mu (\partial_\mu + i e_0 A_\mu^0) \psi_0, \tag{2}$$

$$\mathcal{L}_{\rm GL} = \frac{1}{4} F_{\mu\nu}^2 + |(\partial_\mu - i e_0 A_\mu^0)\phi_0|^2 + m_0^2 |\phi_0|^2 + \frac{u_0}{2}|\phi_0|^4, \tag{3}$$

where the subscript zero denotes bare quantities and $F_{\mu\nu} = \partial_\mu A_\nu^0 - \partial_\nu A_\mu^0$. The labels for the $N_{\rm F}$ fermion and N boson replica are suppressed. We use four-component Dirac fermions which conserve parity and time reversal in three dimensions.

The fermions have the effect of modifying the gauge field properties of the GL model by giving it an effective non-local gradient energy. Indeed, integrating out the fermions generates a leading long-wavelength energy

$$\mathcal{L}_{\rm eff} = \frac{N_{\rm F}}{16} F_{\mu\nu} \frac{1}{\sqrt{-\partial^2}} F_{\mu\nu}. \tag{4}$$

In the infrared, this leading term makes the initial Maxwell term in (3) irrelevant. Since (4) gives the gauge field a unit dimension instead of the canonical 1/2, the charge becomes effectively dimensionless. By integrating out the gauge field for a uniform order field $\phi_0 = \bar{\phi}$, we obtain the effective potential:

$$V_{\rm eff} = \left(m_0^2 + \frac{2e_0^2\Lambda^2}{3\pi^2 N_{\rm F}}\right)|\bar{\phi}|^2 + \left(\frac{u_0}{2} - \frac{32 e_0^4 \Lambda}{3\pi^2 {N_{\rm F}}^3}\right)|\bar{\phi}|^4 - \frac{256 e_0^6}{3\pi^2 {N_{\rm F}}^3}|\bar{\phi}|^6 \ln\left(\frac{8e_0^2|\bar{\phi}|^2}{N_{\rm F}\Lambda}\right), \tag{5}$$

where Λ is an ultraviolet cutoff. Note the important difference with respect to the effective potential of the usual GL model, where the last term has a power $|\phi|^3$ in three dimensions, giving rise to an apparent first-order transition. The limit $N_{\rm F} \to 0$ is singular in this approximation, which ignores the Maxwell term controlling the gauge field fluctuations for $N_{\rm F} = 0$ which produce the $|\phi|^3$ term.

For large $N_{\rm F}$, $V_{\rm eff}$ reduces to the mean-field effective potential of the pure $|\phi|^4$ theory. This decoupling comes from the rescaling the charge $e_0 \to e_0/\sqrt{N_{\rm F}}$ which leads for large $N_{\rm F}$ to an extreme type II superconductor coinciding with the $O(N)$ model. Thus, $N_{\rm F}$ Dirac fermions allows a novel interpolation between the usual GL model and the $O(N)$ model which runs through different intermediate physical systems than the simple limit $e_0^2 \to 0$. It is therefore an interesting problem to study their effect upon the critical behavior their number $N_{\rm F}$ is varied for fixed N. This is what will be done in this paper using RG techniques. At one loop and in $d = 4 - \epsilon$ dimensions we find that for $N = 2$, which is the physical number for a superconductor, an infrared stable charged fixed point exists for $N_{\rm F} > N_{{\rm F}c} = 3.47$. We repeat the study fixed dimensions $d \in (2, 4)$ where we find that the critical number of fermions for $d = 3$ is almost the same: $N_{{\rm F}c} = 4.44$ such that we can give the scheme-independent estimate $N_{{\rm F}c} \approx 4 \pm 0.5$. Finally, all independent critical exponents will be listet as a function of $N_{\rm F}$.

II. CRITICAL BEHAVIOR IN $d = 4 - \epsilon$ DIMENSIONS

Taking into account the Ward identities due to gauge invariance, the Lagrangians (2) and (3) can be written in terms of renormalized quantities as

$$\mathcal{L}_{\rm F} = Z_\psi \bar{\psi} \gamma_\mu (\partial_\mu + ieA_\mu)\psi, \tag{6}$$

$$\mathcal{L}_{\rm GL} = \frac{Z_A}{4} F_{\mu\nu}^2 + Z_\phi |(\partial_\mu - ieA_\mu)\phi|^2 + Z_m m^2 |\phi|^2 + \frac{Z_u u}{2} |\phi|^4. \tag{7}$$

We define the dimensionless couplings:

$$f \equiv \mu^{-\epsilon} Z_A e_0^2 \qquad g \equiv \mu^{-\epsilon} Z_\phi^2 Z_u^{-1} u_0, \tag{8}$$

and fix the renormalization constants by minimal subtraction of $1/\epsilon^n$ pole terms. The resulting one-loop β-functions are

$$\beta_f = -\epsilon f + \frac{8N_{\rm F} + N}{48\pi^2} f^2, \tag{9}$$

$$\beta_g = -\epsilon g - \frac{3fg}{4\pi^2} + \frac{N+8}{16\pi^2} g^2 + \frac{3}{4\pi^2} f^2. \tag{10}$$

For $N_{\rm F} = 0$ we recover the usual one-loop β-functions of the GL model[1,4]. Note that β_g is unaffected by the fermions, being just the one-loop β-function of the GL model.

The fixed points lie at

$$f^* = \frac{48\pi^2 \epsilon}{8N_{\rm F} + N}, \tag{11}$$

$$g_\pm^* = \frac{\epsilon\pi^2}{2} \frac{576N + 16N^2 + 4608N_{\rm F} + 256NN_{\rm F} + 1024{N_{\rm F}}^2 \pm 16(8N_{\rm F} + N)\sqrt{\Delta}}{8N^2 + N^3 + 128NN_{\rm F} + 16N^2N_{\rm F} + 512{N_{\rm F}}^2 + 64N{N_{\rm F}}^2}, \tag{12}$$

where

$$\Delta = -2160 - 360N + N^2 + 576N_{\rm F} + 16NN_{\rm F} + 64{N_{\rm F}}^2. \tag{13}$$

Accessible charged fixed points are obtained only if $\Delta > 0$. The case of interest for superconductivity is $N = 2$ for which Eq. (13) gives, under the condition $\Delta > 0$, a charged fixed point if

$$N_{\rm F} > N_{{\rm F}c} = \frac{6\sqrt{30} - 17}{4} \approx 3.47. \tag{14}$$

We see that the number of fermions does not need to be large in order to produce charged fixed points. A schematic flow diagram is shown in Fig. 1. Remarkably, it exhibits precisely the fixed point structure expected for a GL model[4,8,14]. It has an infrared stable fixed point at (g_+^*, f^*), labeled 'SC' in the figure, which governs the superconducting phase transition. The zero charge nontrivial fixed point labeled 'XY' governs the superfluid He^4 transition with XY critical exponents. This fixed point is unstable for arbitrarily small charge. There is a second charged fixed point labeled 'T' which is infrared stable only along the line flowing from the Gaussian fixed point to it. This fixed point is called the *tricritical* fixed point and the line of infrared stability is a *tricritical line*. The tricritical fixed point has coordinates (g_-^*, f^*). The tricritical line separates the left-hand region where the phase transition is first-order from the

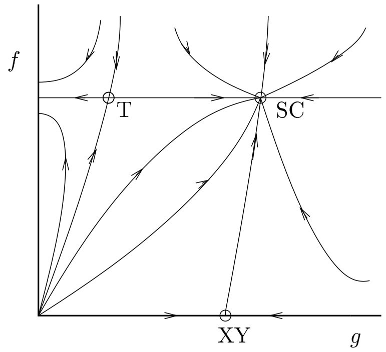

FIG. 1: Schematic flow diagram for the cases where the N_{F} and N values are such that charged fixed are generated.

right-hand region where it is second-order. The tricritical fixed point in the flow diagram is consistent with the proposed phase diagram obtained from duality arguments, where the existence of a tricritical point was predicted by one of us[11] in 1982, and recently confirmed by Monte Carlo simulations[12,13].

Let us compute the anomalous dimension for the case $N_{\mathrm{F}} = 4$ and $N = 2$. This is given the fixed-point value of the RG function

$$\gamma_\phi = \mu \frac{\partial \ln Z_\phi}{\partial \mu}, \tag{15}$$

yielding

$$\eta_\phi \equiv \gamma_\phi(f^*, g_+^*) = -\frac{9\epsilon}{17} \approx -0.53\epsilon. \tag{16}$$

Remakably, the anomalous dimension η_ϕ is *negative* as in the GL model, where it was for a long time a great puzzle, explained only recently as a consequence of momentum space instabilities in the order field correlation function[7,15,16].

In order to evaluate a second critical exponent such as ν we need another RG function

$$\gamma_m = \mu \frac{\partial}{\partial \mu} \ln \left(\frac{Z_m}{Z_\phi} \right). \tag{17}$$

At one-loop order, γ_m is found to be

$$\gamma_m = \frac{1}{16\pi^2}[6f - (N+2)g]. \tag{18}$$

The critical exponent ν is obtained from the infrared stable fixed point value of the function

$$\nu_\phi = \frac{1}{2 + \gamma_m}. \tag{19}$$

Expanded to order ϵ with $N = 2$ and, say, $N_{\mathrm{F}} = 4$, this becomes

$$\nu = \frac{1}{2} + 0.01\epsilon + \mathcal{O}(\epsilon^2). \tag{20}$$

In three dimensions, $\epsilon = 1$ such that $\nu \approx 0.51$.

Below T_c, the gauge field acquires a mass whose inverse is the penetration depth λ. The ratio between the λ and ξ defines the Ginzburg parameter κ. Its square can be expressed in term of the coupling constants as $\kappa^2 = g/2f$. At the mean-field leve,l type I and type II superconductivity are observed for $\kappa < 1/\sqrt{2}$ and $\kappa > 1/\sqrt{2}$, repsectively. Fluctuations will renormalize this separation point to $\kappa_-^{*2} \equiv g_-^*/2f^*$. The value of κ^2 at the superconducting fixed point is given by $\kappa_+^{*2} = g_+^*/2f^*$. For $N = 2$ and $N_{\mathrm{F}} = 4$, we have

$$\kappa_-^* = \frac{1.24}{\sqrt{2}}, \qquad \kappa_+^* = \frac{1.77}{\sqrt{2}}. \tag{21}$$

Both values are *above* the mean-field value $1/\sqrt{2}$, in contrast to the theoretical[11] and the Monte Carlo numbers[12,13] in the GL model.

In Table I we show the values of critical exponents and the Ginzburg parameter for $N = 2$ and $\epsilon = 1$ for growing values of N_{F}. We see that both anomalous dimensions approach zero as N_{F} becomes large, η_ψ from positive values, and η_ϕ from negative values. The anomalous dimensions tend to zero with increasing N_{F} much more rapidly than the other quantities in Table I. Note that κ_-^* decreases with N_{F} while κ_+^* increases with N_{F}. Interestingly, the critical exponent ν does not change very much with N_{F}, attaining quickly a limit value $\nu \approx 0.6$.

The limit $N_{\mathrm{F}} \to \infty$ at fixed N can be done analytically in the above equations:

$$\lim_{N_{\mathrm{F}} \to \infty} f^*|_{N=2} \to 0, \qquad \lim_{N_{\mathrm{F}} \to \infty} g_-^*|_{N=2} \to 0, \tag{22}$$

$$\lim_{N_{\mathrm{F}} \to \infty} g_+^*|_{N=2} \to \frac{8\pi^2}{5}. \tag{23}$$

This implies that for many fermions, $\kappa_+^* \to \infty$ while $\kappa_-^* \to 0$ for $N = 2$. The limiting critical exponents are $\eta_\psi = \eta_\phi = 0$ and $\nu = 0.6$. Since $\kappa_+^* \to \infty$ as $N_{\mathrm{F}} \to \infty$ at $N = 2$ fixed, this limit is an extreme type II limit in our model. Remarkably, the limiting value $\nu = 0.6$ equals the one-loop result for the XY-model in the ϵ-expansion[2].

III. CRITICAL BEHAVIOR IN FIXED DIMENSIONS $d \in (2, 4)$

Let us compare the result in $d = 4 - \epsilon$ dimensions to the fixed dimension RG approach. Instead computing the β-functions for ϵ small, we can set $m^2 = 0$ and compute

TABLE I: Critical exponents and values of the Ginzburg parameter κ at the tricritical and superconducting fixed point for $\epsilon = 1$ and $N = 2$ for several values of $N_{\rm F}$.

$N_{\rm F}$	η_ϕ	ν	$\kappa_-^*(T)$	$\kappa_+^*(SC)$
4	-0.53	0.51	$1.24/\sqrt{2}$	$1.77/\sqrt{2}$
5	-0.43	0.54	$1.09/\sqrt{2}$	$2/\sqrt{2}$
6	-0.36	0.55	$1.01/\sqrt{2}$	$2.17/\sqrt{2}$
10	-0.21	0.58	$0.82/\sqrt{2}$	$2.68/\sqrt{2}$
15	-0.15	0.59	$0.69/\sqrt{2}$	$3.17/\sqrt{2}$
20	-0.11	0.6	$0.61/\sqrt{2}$	$3.58/\sqrt{2}$
100	-0.02	0.6	$0.29/\sqrt{2}$	$7.47/\sqrt{2}$
1000	-0.002	0.6	$0.1/\sqrt{2}$	$23.19/\sqrt{2}$
10000	-0.0002	0.6	$0.03/\sqrt{2}$	$72.57/\sqrt{2}$

TABLE II: Critical exponents and values of the Ginzburg parameter at the tricritical and superconducting fixed points for $d = 3$ and $N = 2$ for several values of $N_{\rm F}$.

$N_{\rm F}$	η_ϕ	ν	κ_-^*	κ_+^*
5	-0.36	0.53	$1.13/\sqrt{2}$	$1.6/\sqrt{2}$
6	-0.31	0.55	$1/\sqrt{2}$	$1.79/\sqrt{2}$
10	-0.19	0.59	$0.79/\sqrt{2}$	$2.28/\sqrt{2}$
15	-0.13	0.6	$0.65/\sqrt{2}$	$2.7/\sqrt{2}$
20	-0.1	0.61	$0.6/\sqrt{2}$	$3.08/\sqrt{2}$
100	-0.02	0.623	$0.27/\sqrt{2}$	$6.44/\sqrt{2}$
1000	-0.002	0.625	$0.09/\sqrt{2}$	$20.06/\sqrt{2}$
10000	-0.0002	0.625	$0.03/\sqrt{2}$	$63.25/\sqrt{2}$

the Feynman integrals for any dimension $d \in (2,4)$. The β-functions are in this case given at one loop by

$$\beta_f = (4-d)\{-f + [8N_{\rm F}A(d) + NB(d)]f^2\}, \tag{24}$$

$$\beta_g = (4-d)\Bigg\{ -g + C(d) \times \left[-2(d-1)f + \frac{N+8}{2}g^2 + 2(d-1)f^2\right]\Bigg\}, \tag{25}$$

where

$$A(d) = \frac{\Gamma(2-d/2)\Gamma^2(d/2)}{(4\pi)^{d/2}\Gamma(d)}, \tag{26}$$

$$B(d) = -\frac{\Gamma(1-d/2)\Gamma^2(d/2)}{(4\pi)^{d/2}\Gamma(d)}, \tag{27}$$

$$C(d) = \frac{\Gamma(2-d/2)\Gamma^2(d/2-1)}{(4\pi)^{d/2}\Gamma(d-2)}. \tag{28}$$

The RG functions γ_ϕ and γ_m, are at one loop:

$$\gamma_\phi = (1-d)(4-d)C(d)f, \tag{29}$$

$$\gamma_m = (N+2)(d-4)C(d)g/2 - \gamma_\phi. \tag{30}$$

Since we are working at the critical point, the RG function γ_m above is obtained from an insertion of the composite field $|\phi|^2$ into the two-point function.

As a cross check we set $d = 4-\epsilon$ and expand to first order in ϵ, and verify that the β-functions (24) and (25) reduce correctly to the previous (9) and (10), respectively.

In the absence of fermions, the critical value of N above which charged fixed points exist for $d = 3$ is $N_c = 103.4$, much smaller than the value given in the ϵ-expansion, $N_c = 365.9$. On the other hand, when we set $N = 2$ the critical number of fermions is larger than in the ϵ-expansion, being given by $N_{{\rm F}c} = 4.44$. On the basis of the result of Section II, we can gige the scheme independent estimate as $N_{{\rm F}c} = 4 \pm 0.5$.

In Table II we show the values of critical exponents and Ginzburg parameter at $d = 3$ and $N = 2$ for several values of $N_{\rm F}$. Qualitatively we observe the same behavior as in Table I.

The large $N_{\rm F}$ limit at fixed $N = 2$ and $d = 3$ gives $f^* = 0$ and

$$\lim_{N_{\rm F}\to\infty} g_+^*|_{N=2} \to \frac{8}{5}. \tag{31}$$

Thus, we obtain the large $N_{\rm F}$ value of ν at $N = 2$ and $d = 3$:

$$\nu = \frac{5}{8} \approx 0.625. \tag{32}$$

As in Section II, we have $\eta_\psi = \eta_\phi = 0$ at $N_{\rm F} = \infty$. We obtain a critical exponent ν at large $N_{\rm F}$ and $N = 2$ corresponding to the one-loop value of the XY-model obtained in the fixed dimension approach[17].

IV. DISCUSSION

We have shown that $N_{\rm F}$ additonal Dirac fermions in a GL model lead to a charged fixed point for $N_{\rm F} \geq 4$. In the limit $N_{\rm F} \to \infty$, the model approaches the extreme type II limit of superconductivity. The fermionic Lagrangian $\mathcal{L}_{\rm F}$ can thus be used to control the influence of the thermal magnetic fluctuations, suppressing them with increasing $N_{\rm F}$. An interesting physical case in $d = 3$ dimension has fermion number $N_{\rm F} = 10$, where the critical exponents and values of κ at the tricritical and superconducting fixed points are close to the predicted[11] and Monte Carlo -measured values[13,18,19] for the pure GL model. The critical exponent ν obtained from Monte Carlo simulations[18] has the XY model value $\nu \simeq 0.67$, as predicted[20] from disorder field theory of superconductors[21]. The anomalous dimension η_ϕ obtained from Monte Carlo simulations is[19] $\eta_\phi \simeq -0.18$. The value of κ at the tricritical point predicted from the disorder field theory is[11]

$\kappa_-^* = 0.79/\sqrt{2}$, confirmed by recent the Monte Carlo simulations giving $\kappa_-^* = 0.76/\sqrt{2}$.[13] The results in Table II, show amazing agreement of the theoretical values of η_ϕ and κ_-^* in the present kodel with $N_{\rm F} = 6$. The critical exponent $\nu = 0.59$ deviates slightly from the one-loop XY result, but the difference is not excessive for a one-loop result.

For $N_{\rm F} < 4$ the model has no second-order phase transition but interesting physical effects can be expected due to chiral symmtery breaking. At $d = 3$ the Lagrangian (1) has a chiral symmetry[22] $\psi \to \exp(i\gamma_{3,5}\theta)\psi$, with

$$\gamma_3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

where I is a 2×2 unit matrix. In the absence of scalar bosons, this symmetry is spontaneously broken for $N_{\rm F} < 32/\pi^2 \approx 3.24$ and a fermion mass is dynamically generated.[23] According to Kim and Lee[10], coupling to bosons reduce this upper bound by factor of two and one has $N_{\rm F} < 16/\pi^2 \approx 1.62$. This lies below the critical value $N_{{\rm F}c} = 4.44$ for the existence of charged fixed points. The critical behavior described in this paper is thus apparently not affected by the chiral symmetry breaking. However, we may wonder if the dynamical mass generation in the chirally symmetry-broken phase can generate new fixed points in our system.

---

* Electronic address: `kleinert@physik.fu-berlin.de`; URL: `http://www.physik.fu-berlin.de/~kleinert/`

† Electronic address: `nogueira@physik.fu-berlin.de`

1. B. I. Halperin, T. C. Lubensky and S.-K. Ma, Phys. Rev. Lett. **32**, 292 (1974); J.-H. Chen, T. C. Lubensky and D. R. Nelson, Phys. Rev. B **17**, 4274 (1978).
2. H. Kleinert and V. Schulte-Frohlinde, *Critical Phenomena in Φ^4-Theory*, World Scientific, Singapore 2001 (`http://www.physik.fu-berlin.de/~kleinert/b8`).
3. S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973). For a comparison of the two theories see H. Kleinert, Phys. Lett. B **128**, 69 (1983).
4. I. D. Lawrie, Nucl. Phys. B **200**, 1 (1982).
5. J. Tessmann, MS thesis 1984 written under the supervision of one the present authors (HK); the pdf file is accessible at internet `www.physik.fu-berlin.de/~kleinert/MS-Tessmann.pdf`.
6. S. Kolnberger and R. Folk, Phys. Rev. B **41**, 4083 (1990).
7. H. Kleinert and F. S. Nogueira, `cond-mat/0104573`.
8. I. F. Herbut and Z. Tešanović, Phys. Rev. Lett. **76**, 4588 (1996).
9. J. B. Marston and I. Affleck, Phys. Rev. B **39**,11538 (1989); J. B. Marston, Phys. Rev. Lett. **61**, 1914 (1988).
10. D. H. Kim and P. A. Lee, Ann. Phys. (N.Y.) **272**, 130 (1999).
11. H. Kleinert, Lett. Nuovo Cimento **35**, 405 (1982).
12. J. Bartholomew, Phys. Rev. B **28**, 5378 (1983).
13. S. Mo, J. Hove, A. Sudbø, *The order of the metal to superconductor transition*, cond-mat/0109260.
14. R. Folk and Y. Holovatch, J. Phys. A **29**, 3409 (1996).
15. F. S. Nogueira, Phys. Rev. B **62**, 14559 (2000).
16. J. Hove and A. Sudbø, Phys. Rev. Lett **84**, 3426 (2000).
17. C. Itzykson and J.-M. Drouffe, *Statistical Field Theory* Vol. 1 (Cambridge University Press, Cambridge, 1989).
18. P. Olsson and S. Teitel, Phys. Rev. Lett. **80**, 1964 (1998).
19. A. K. Nguyen and A. Sudbø, Phys. Rev. B **60**, 15307 (1999).
20. M. Kiometzis, H. Kleinert and A. M. J. Schakel, Phys. Rev. Lett. **73**, 1975 (1994) (`cond-mat/9503019`); Fortschr. Phys. **43**, 697 (1995) (`cond-mat/9508142`).
21. H. Kleinert *Gauge Fields in Condensed Matter*, Vol. I Superflow and Vortex Lines, World Scientific, Singapore 1989, pp. 1–744, (`http://www.physik.fu-berlin.de/~kleinert/b1`)
22. R. Pisarski, Phys. Rev. D **29**, 2423 (1984).
23. T. W. Appelquist, M. Bowick, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. D **33**, 3704 (1986).