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ELECTROMAGNETIC COUPLINGS FROM THE COMBINED USE OF FORWARD AND BACKWARD DISPERSION RELATIONS ‡

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Abstract: We derive sum rules for the isovector electromagnetic transition moments $N^{**} \to N^{*}\gamma$ where N^{*}, N^{**} are arbitrary resonances. The derivation is based on the following assumptions: (i) the isospin odd amplitudes of the scattering process $N^{**}\pi \leftarrow N^{*}\pi$ obey unsubstracted dispersion relations in forward as well as in backward direction; (ii) the coupling constants of the s channel resonances are given, to a good approximation, by existing solutions of the algebra of axial charges; (iii) the *t*-channel contribution of these amplitudes is dominated by a single ρ meson; and (iv) the $N^{**}N^{*}\rho$ coupling constant can be related to the electromagnetic couplings $N^{**}N^{*}\gamma$ via vector meson dominance.

Our equations are tested by a calculation of the known couplings of the nucleon and the $\Delta N\gamma$ transition. The results agree well with experiment. We then *predict* the isovector couplings

$$R(1470)\,N\gamma\,(K^{V}_{RN} \approx 1.2), \qquad R(1470)\,R\gamma\,(K^{V}_{RR} \approx 1.3)\ ,$$

where R(1470) is the $\frac{1}{2}^{+}$ Roper resonance. We conclude that R(1470) should be seen in photoproduction on deuteron.

1. INTRODUCTION

In the past, sum rules on scattering amplitudes have been of great help in gaining some insight in the interrelation of coupling strenghts of baryon and meson resonances. An important class of such sum rules is obtained by combining the knowledge of some low energy value of an amplitude, taken from a symmetry principle, with an unsubstracted dispersion relation and saturating the dispersion integral with a finite number of resonances. In this manner, the low energy theorems supplied by current algebra yield, via the PCAC hypothesis, a large set of Adler-Weisberger relations [1] involving coupling constants of pions to arbitrary resonances. They can be

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solved most efficiently by algebraic methods [2--4]. Similarly, for Compton scattering one obtains the Drell-Hearn [4] and Cabibbo-Radicati [5] type of sum rules.

If no low energy theorems are available from symmetry priciples, other information is needed to determine at least one value of an amplitude. This information may come from a knowledge of the asymptotic behaviour. For example if an amplitude is known to converge asymptotically to zero by one power stronger than required for an unsubtracted dispersion relation, one can convert this information into so called superconvergence sum rules. Similarly, if the asymptotic behaviour is controlled by a Regge formula one can derive the well known finite energy sum rules.

A further possibility for deriving sum rules opens up if there is more than one independent variable in which an amplitude obeys unsubtracted dispersion relations. It is the purpose of this paper to describe the sum rules that emerge in this manner if an amplitude obeys unsubtracted dispersion relations for scattering in the forward as well as in the backward direction.

Backward dispersion relations have first been used in πN elastic scattering in order to obtain information on the $\pi\pi$ scattering phase shifts [6]. A little later it was recognized that by equating the values of pion nucleon amplitudes at threshold obtained by dispersing in the backward direction with those obtained from the forward dispersion relation leads to sum rules connecting s- and t- channel absorptive parts [7]. These sum rules were studied by Höhler and collaborators for the isospin odd [8] as well as for the isospin even [9] amplitudes. It turns out that the t- channel cut of the isospin odd amplitude can be approximated quite well by a simple ρ pole. The coupling constants of ρ to nucleons can be obtained and agree reasonably with those obtained from vector meson dominance. For the isospin even amplitudes this approach lead to first estimates on the coupling of the partly hypothetical σ particle. (m_σ = 700 MeV, $\Gamma_{\sigma\to\pi\pi}$ = 400 MeV) [9, 10] and of the f meson (m_{f} = 1260 MeV, $\Gamma_{\mathrm{f}\to\pi\pi}$ = 150 ± 25 MeV) to nucleons. The f was found to decouple from the flip amplitude of the nucleon [9]. This is in good agreement with the observation that the pomeron and f trajectories apparently decouple from the nucleon helicity flip amplitude [11] thus exhibiting an extreme smoothness of the Regge residue of the f trajectory. Guided by these results on πN scattering we propose to study the possibility of determining the electromagnetic coupling of arbitrary resonances by the same method.

The program is as follows. From the Regge pole hypotheses we can estimate the asymptotic behaviour of the invariant amplitudes for the process $\pi N^{**} \leftarrow \pi N^*$ (N*, N** some nucleon excited states) in forward as well as in backward direction. If an amplitude is found to converge to zero in either case we disperse in both directions and equate the threshold values. We saturate the resulting sum rules by the important resonances in the s-channel and by a single ρ meson in the t-channel. The s-channel coupling constants for higher resonances are in general unknown experimentally. Therefore we shall use as an approximation the values obtained from algebraic solutions of Adler Weisberger relations [2, 3]. In this way one obtains the couplings of the ρ meson to the baryon resonances. One then invokes the hypothesis of vector meson dominance of the electromagnetic current to relate the ρ meson coupling to the photon vertex. We shall assume the external pion mass to be continued to zero in our discussion. This allows for a considerable simplification of kinematics and is known, due to PCAC, to introduce errors not larger than 8%.

2. GENERAL FRAMEWORK

Consider the scattering of zero mass pions on arbitrary targets m and m'. Then the scattering angle in the c.m.s. of the s-channel is given by

$$\sin\theta_s = 2\sqrt{s}\sqrt{\phi(s,t)}\,/\,(s_{12}\,s_{34}) \ , \tag{2.1}$$

$$\cos\theta_s = [2st + s^2 - s\,(m^2 + m'^2) + m^2 m'^2]/\,(s_{12}\,s_{34}) \ , \tag{2.2}$$

with

$$\phi(s,t) = t(su - m^2 m'^2) = -t(ts + (s-m^2)\,(s-m'^2)) \ , \tag{2.3}$$

$$s_{12} = s - m^2, \qquad s_{34} = s - m'^2 \ . \tag{2.4}$$

Thus forward scattering occurs along the line $t = 0$, while the backward direction is defined by

$$su - m^2 m'^2 = 0 \ . \tag{2.5}$$

Consider an amplitude $A(s,t,u)$ with the following properties:

(i) $A(s,t,u)$ is crossing even under the exchange $s \leftrightarrow u$,

(ii) $A(s,t,u)$ obeys a Mandelstam respresentation,

(iii) $A(s,t,u)$ tends to zero for $s \to \infty$ and $\theta_s = 0$ as well as $\theta_s = 180°$.

Then $A(s,t,u)$ satisfies an unsubtracted dispersion relation at $t = 0$ in $\nu = \frac{1}{2}(s-u)$ of the form

$$A(\nu^2,\mathrm{o}) = \frac{1}{\pi}\int \frac{\mathrm{Im}A(\nu'^2,\mathrm{o})}{\nu'^2 - \nu^2}\,\mathrm{d}\nu' \ . \tag{2.6}$$

In the backward direction we choose t as the variable in which to disperse and note that (2.5) leads to

$$\left\{ {s \atop u} \right\} = -\tfrac{1}{2}(t - m^2 - m'^2) \pm \tfrac{1}{2}\sqrt{(t-(m+m')^2)\,(t-(m-m')^2)} \ , \tag{2.7}$$

which implies kinematic singularities of s and u and consequently for $A(s,t,u)$. An even amplitude, however, depends only on ν^2 which in the backward directions becomes

$$\nu_B^2 = \tfrac{1}{4}(t-(m+m')^2)(t-(m-m')^2) \tag{2.8}$$

and can therefore be dispersed in t without any problem. Notice that it is for this point that the approximation of zero pion mass was made. For nonzero pion mass (2.7) would read

$$\left\{{s \atop u}\right\} = \tfrac{1}{2}(t-m^2-m'^2-2\mu^2) \pm \tfrac{1}{2}\sqrt{\frac{t-4\mu^2}{t}(t-(m'+m)^2)(t-(m'-m)^2)}\ ,$$

thus introducing a kinematic pole at $t = 0$ in ν^2. Only if $m = m'$ does this pole disappear and we can in that case relax the condition $\mu^2 = 0$.

The backwards dispersion relation for $A(\nu^2,t)$ reads

$$A(\nu_B^2(t),t) = \frac{1}{\pi}\int \frac{\mathrm{Im}A(\nu'^2_B(t'),t')}{t'-t}\,\mathrm{d}t'\ . \tag{2.9}$$

Equating (2.6) and (2.9) at $\nu = \frac{1}{2}(m'^2-m^2)$, $t = 0$ we obtain

$$\frac{1}{\pi}\int\frac{\mathrm{Im}A(\nu'^2,\mathrm{o})}{\nu'^2}\,\mathrm{d}\nu' - \frac{1}{\pi}\int\frac{\mathrm{Im}A(\nu'^2(t'),t')}{t'}\,\mathrm{d}t' = \mathrm{o}\ . \tag{2.10}$$

Consider now the contribution of an s-channel resonance of mass m_n in the sharp resonance approximation. Its contribution to $A(\nu^2,t)$ is

$$[A(\nu^2,t)]_n = R_n^s(g_{nm'}g_{m'n},m_n^2,t)\left[\frac{1}{s-m_n^2}+\frac{1}{u-m_n^2}\right]\ . \tag{2.11}$$

Here $R_n^s(g_{nm}, g_{m'n}, m_n^2, t)$ is linear combination of products of coupling constants $g_{nm}\,g_{m'n}$ with kinematical and angular momentum factors. Their detailed structure will be specified below. The contribution to the forward dispersion integral in (2.10) is then

$$[A_F(\mathrm{o},\mathrm{o})]_n = R_n^s(g_{nm}, g_{m'n}, m_n^2, t)\ \frac{m^2+m'^2-2m_n^2}{(m^2-m_n^2)(m'^2-m_n^2)}\ . \tag{2.12}$$

To obtain the contribution to the backward dispersion integral we note that the square bracket in (2.11) can be written as

$$\left[\frac{1}{s-m_n^2}+\frac{1}{u-m_n^2}\right]=\frac{-t+m^2+m'^2-2m_n^2}{(s-m_n^2)\,(u-m_n^2)}=\frac{-t+m^2+m'^2-2m_n^2}{(t-t_n)\,m_n^2}\;, \tag{2.13}$$

where

$$t_n=-\frac{1}{m_n^2}(m_n^2-m^2)\,(m_n^2-m'^2) \tag{2.14}$$

is the t value at which the resonance leads to a pole in backward direction. The expression (2.13) becomes unsubstracted if t is replaced by t_n in the numerator. Then the contribution to the backward dispersion integral in (2.10) is

$$[A_B\,(\mathrm{o,o})]_n=R_n^s\,(g_{nm},g_{m'n},m_n^2,t_n)\,\frac{m^2m'^2-m_n^2}{m_n^2\,(m^2-m_n^2)\,(m'^2-m_n^2)} \tag{2.15}$$

The t-channel resonance contributions are of the form

$$[A\,(\nu^2,t)]_r=R_r^t\,(g_{m'mr},g_{r\pi\pi},m_r^2,\theta_t)\,\frac{1}{t-m_r^2} \tag{2.16}$$

and therefore yield at threshold

$$[A_B\,(\mathrm{o,o})]_r=-R_r^t\,(g_{m'mr},g_{r\pi\pi},m_r^2,\pi)\,\frac{1}{m_r^2}\,. \tag{2.17}$$

As a final result we obtain the sum rule

$$-\sum_r R_r^t\,(g_{m'mr},g_{r\pi\pi},m_r^2,\pi)\,\frac{1}{m_r^2}=-\sum_n\left\{R_n^s\,(g_{m'n},g_{nm},m_n^2,\mathrm{o})\,(2m_n^2-m^2-m'^2)\right.$$

$$\left.-R_n\,(g_{m'n},g_{nm},m_n^2,t_n)\,\frac{m_n^4-m^2m'^2}{m_n^2}\right\}\frac{1}{(m_n^2-m^2)\,(m_n^2-m'^2)}\,. \tag{2.18}$$

As we said in the introduction, we shall use for the coupling constants $g_{m'n}, g_{nm}$ the values obtained from an algebraic solution of Adler Weisberger relations [2, 3]. In order to do so we relate the invariant functions free of kinematical singularities and constraints $A(s,t)$ to s-channel helicity amplitudes

$$\hat{T}^{s}_{\lambda'\lambda} = (\cos\tfrac{1}{2}\,\theta s)^{-|\lambda+\lambda'|}\,(\sin\tfrac{1}{2}\,\theta s)^{-|\lambda'-\lambda|}\,T^{s}_{\lambda'\lambda}$$

as

$$A(s,t) = \sum_{\lambda,\lambda'} M_{\lambda'\lambda}(s,t)\,\hat{T}^{s}_{\lambda'\lambda}\ . \tag{2.19}$$

Close to a sharp resonance of mass m_n and spin J_n in the s-channel the helicity amplitudes behave as

$$\hat{T}^{s}_{\lambda'\lambda} = \frac{1}{s-m_n^2}\, j_b(\lambda')_{m'n}\, j_a(\lambda)_{nm}\, \hat{\mathrm{d}}^{J_n}_{\lambda\lambda'}(\theta_s)\ , \tag{2.20}$$

where $\hat{d}^{J}_{\lambda\lambda'}$ are related to the usual rotation matrices $d^{J}_{\lambda\lambda'}$ in the same way as $\hat{T}^{s}_{\lambda\lambda'}$ to $T^{s}_{\lambda\lambda'}$. The $j_a(\lambda)_{m'm}$ are the matrix elements of the pionic current with the isospin label $a(a = 1, 2, 3)$ between two states of helicity λ^{+}. Since the mass of the external pion is assumed to be continued to zero we can use PCAC to express $j_a(\lambda)_{m'm}$ in terms of the invariant collinear matrix elements X of A^{0+3}

$$\begin{aligned} j_a(\lambda)_{m'm} &= \langle p'_z,m',\lambda|j_a(0)|p_z,m,\lambda\rangle \\ &= \frac{1}{F_\pi}\langle p'_z,m',\lambda|\partial_\mu A^{\mu}_{a}|p,m,\lambda\rangle \\ &= \frac{i}{F_\pi}(p'-p)^0\langle p'_z,m',\lambda|A^{0+3}_{a}|p_z,m,\lambda\rangle \\ &= \frac{i}{F_\pi}(m'^2-m^2)\frac{1}{2(p_0+p_3)}\langle p'_z,m',\lambda|A^{0+3}_{a}|p_z,m,\lambda\rangle \\ &\equiv \frac{i}{F_\pi}(m'^2-m^2)X_a(\lambda)\ . \end{aligned} \tag{2.21}$$

In terms of these matrix elements, T becomes

$$\hat{T}^{s}_{\lambda'\lambda} = \frac{1}{s-m_n^2}\,\frac{1}{F_\pi^2}(m'^2-m_n^2)(m^2-m_n^2)X_b(\lambda')X_a(\lambda)\,\hat{d}_{\lambda\lambda'}(\theta) \tag{2.22}$$

+ Normalized to $2\,p_0(2\pi)^3\delta^3(p-p')$.

which can be inserted into (2.19) and compared with (2.11) to obtain

$$R(g_{m'n}, g_{nm}, m_n^2, t) = \sum_{\lambda\lambda'} M_{\lambda'\lambda}(m_n^2) \frac{1}{F_\pi^2} (m'^2 - m_n^2)(m^2 - m_n^2) \times X_b(\lambda') X_a(\lambda) \; .$$

Then the sum rule (2.18) becomes

$$- \sum_r R_r^t (g_{m'mr}, g_{r\pi\pi}, m_r^2, \pi) \frac{1}{m_r^2} = - \frac{1}{F_\pi^2} \sum_{\lambda\lambda'} [M_{\lambda'\lambda}(m_r^2, 0)(2m_n^2 - m^2 - m'^2) d^{J_n}_{\lambda\lambda'}(0)$$

$$-M_{\lambda'\lambda}(m_r^2, t_n) \frac{m_n^4 - m^2 m'^2}{m_n^2} d^{J_n}_{\lambda\lambda'}(\pi)] \, X_b(\lambda') \, X_a(\lambda) \; . \tag{2.23}$$

For the rotation matrices $d^{J_n}_{\lambda\lambda'}$ one finds explicitly for $\lambda > \lambda'$

$$d^{J_n}_{\lambda\lambda'}(0) = (-1)^{\lambda - \lambda'} d^{J_n}_{\lambda'\lambda}(0) = (-1) \frac{1}{(\lambda - \lambda')!} \sqrt{\frac{(J_n + \lambda)! \, (J_n - \lambda')!}{(J_n + \lambda')! \, (J_n - \lambda)!}} \; ,$$

$$d^{J_n}_{\lambda\lambda'}(\pi) = (-1)^{J_n - \lambda'} d^{J_n}_{-\lambda'\lambda}(0) = (-1)^{J_n + \lambda} d^{J_n}_{\lambda, -\lambda'}(0) \; . \tag{2.24}$$

The matrix elements $X_a(\lambda)$ are known to satisfy, together with the isospin operators T_a, the algebra of SU(2) × SU(2), i.e.

$$[T_a, T_b] = i\epsilon_{abc} T_c, \qquad [T_a, X_b(\lambda)] = i\epsilon_{abc} X_c(\lambda), \qquad [X_a(\lambda), X_b(\lambda)] = i\epsilon_{abc} T_c \; . \tag{2.25}$$

From the definition (2.21) it also follows that $X(\lambda)$ satisfies

$$X(\lambda)_{m'm} = -\eta_{m'} \eta_m (-1)^{J_{m'} - J_m} X(-\lambda) \tag{2.26}$$

where η_m denotes the parity of particle m. If we define reduced matrix elements of X by [33] +

$$X_a(\lambda)_{m'm} = G_A \chi^+_{m'} \frac{\tau_a}{2} \chi_m \; , \qquad (\tfrac{1}{2} \leftrightarrow \tfrac{1}{2}) \; , \tag{2.27a}$$

$$X_a(\lambda)_{m'm} = \frac{\sqrt{3}}{2} G^* \chi^+_a \chi, \qquad (\tfrac{3}{2} \leftrightarrow \tfrac{1}{2}) \; , \tag{2.27b}$$

+ χ are $T = \frac{1}{2}$, $\chi_a = \sqrt{\frac{3}{2}} \, (e_a - \frac{1}{3} \tau_a (\tau e)) \, \chi$ are $T = \frac{3}{2}$ isospinors.

$$X_a(\lambda)_{m'm} = -\tfrac{3}{2}\, iG^*_{\rm A}\epsilon_{\rm abc}\chi^+_{\rm b}\chi_{\rm c}, \qquad (\tfrac{3}{2} \leftrightarrow \tfrac{3}{2}) \;, \tag{2.27b}$$

for the transitions between different isospins, the commutation rules (2.25) are equivalent to the set of Adler Weisberger relations for the scattering of pions on spin $\frac{1}{2}$ and spin $\frac{3}{2}$ targets N and Δ, respectively

$$G^2_{\rm A} - G^{*2} = 1 \;, \qquad (\pi{\rm N} \leftarrow \pi{\rm N}) \;,$$

$$G^{*2}_{\rm A} + \tfrac{1}{2}G^{*2} = 1 \;, \qquad (\pi\Delta \leftarrow \pi{\rm N}) \;,$$

$$G^*G_{\rm A} - 5G^*_{\rm A}G^* = 0 \;, \qquad (\pi\Delta \leftarrow \pi\Delta) \;. \tag{2.28}$$

Here G has to be understood as a matrix in the space of particles of definite isospins.

The simplest nontrivial solutions of these equations are the following

(1) only one N and one Δ particle are present

(a) No NΔ transitions, i.e. $G^* = 0$. Then $G_{\rm A} = 1, G^*_{\rm A} = 1$.

(b) There are NΔ transitions, then $G_{\rm A} = \frac{5}{3}, G^* = \frac{4}{3}, G^*_{\rm A} = \frac{1}{3}$, which is the well known SU(4) solution

(2) Two $I = \frac{1}{2}$ and one $I = \frac{3}{2}$ resonances, the nucleon, the Roper R(1470) and the Δ(1236) are present with one free mixing angle θ. Then

$$G_{\rm A} = \begin{Bmatrix} 1+\frac{2}{3}\cos^2\theta & \frac{2}{3}\cos\theta\sin\theta \\ \frac{2}{3}\cos\theta\sin\theta & 1+\frac{2}{3}\sin^2\theta \end{Bmatrix} ,$$

$$G^* = (\cos\theta, \sin\theta) \;, \qquad G^*_{\rm A} = \tfrac{1}{3} \;. \tag{2.29}$$

For a first rough calculation we shall stick to the last approximation. The angle θ has to be taken around 45° in order to get reasonable agreement with experiment *.

* Experimentally one has $G_{\rm A} \approx 1.23$, $(G_{\rm A})_{\rm RN} \approx 0.37$ from

$$\Gamma_{{\rm R}\to{\rm N}\pi} = \frac{1}{16\pi F^2_\pi}\,\frac{(m^2_{\rm R} - m^2)^3}{m^3_{\rm R}}\,\frac{3}{4}(G_{\rm A})^2_{\rm RN} = (G_{\rm A})^2_{\rm RN} \times 0.86\ {\rm GeV} = 0.120\ {\rm GeV}$$

and $G^* = 1$ from $\Gamma_{\Delta\to{\rm N}\pi} = \dfrac{1}{16\pi F^2_\pi}\,\dfrac{m^2_\Delta - m^2}{m^3_\Delta}\,\dfrac{3}{8}G^{*2} \approx G^{*2} \times 0.123\ {\rm GeV} = 0.120\ {\rm GeV}$.

Notice that in terms of the reduced matrix elements G the contribution of $X_b(\lambda')\,X_a(\lambda)$ to the $I_t = 1$ combinations of A can be written as

$$[X_b(\lambda'), X_a(\lambda)]_{\frac{1}{2}\frac{1}{2}} = \tfrac{1}{2}\,[G(\lambda')\,G(\lambda) - G^*(\lambda')\,G^*(\lambda)]\ \tfrac{1}{2}\ [\tau_b\tau_a]\ ,$$

$$[X_b(\lambda'), X_a(\lambda)]_{\frac{3}{2}\frac{1}{2}} = \tfrac{1}{4}\ [G^*(\lambda')\,G_{\rm A}(\lambda) + 5G^*_{\rm A}(\lambda)] \times (\chi^+_b\,\tau_a\,\chi - \chi^+_a\,\tau_b\,\chi)\ ,$$

$$[X_b(\lambda'), X_a(\lambda)]_{\frac{3}{2}\frac{3}{2}} = \tfrac{1}{4}\ [G^*(\lambda')\,G^*(\lambda) + G^*_{\rm A}(\lambda')\,G_{\rm A}(\lambda)]$$

$$\times\ i\epsilon_{bad}\,(-i\epsilon_{def}\,\chi_e\chi_f)\ ,$$

where the commutation stands for antisymmetrization in b and a.

The construction of the matrix $M_{\lambda'\lambda}(s,t)$ relating the s-channel helicity amplitudes to the invariant amplitudes free of kinematical singularities will have to be done for each process separately.

The contribution of arbitrary t-channel resonances to the sum rule (2.23) can be calculated in a completely analogous fashion. However, since we shall assume a single ρ meson to make up the dominant part of it, we find it more convenient to calculate the residues $R^t_r\,(g_{\rho mm'}, g_{\rho\pi\pi}, m^2_\rho, \theta)$ directly via a Feynman graph for every process under consideration.

We shall therefore turn now directly to the applications.

3. APPLICATION TO R(1470) $\pi \leftarrow$ Nπ

The invariant amplitudes free of kinematical singularities and constraints for the process $(\frac{1}{2}^+)\,(p',m')\,\pi(q') \leftarrow (\frac{1}{2}^+)\,(p,m)\,\pi(q)$ are defined by ‡

$$T = -\bar{u}(p')\,(A + \tfrac{1}{2}\,(\not{q}-\not{q}')\,B)\,u(p)\ \sqrt{4m'm}\ . \tag{3.1}$$

If the incoming $\frac{1}{2}$‡ particle moves as particle 1 in the positive z direction, the s-channel c.m. helicity amplitudes ‡‡ can be expressed in terms of A and B in the form ‡‡‡

‡ Normalization: $S = 1 - i\,(2\pi)^4\,\delta^4\,(P_{\rm f} - P_{\rm i})\,T$.

‡‡ Jacob Wick conventions.

‡‡‡ These are defined by $A_{ba} = \delta_{ba}\boldsymbol{A}^{(+)} + \frac{1}{2}\,[\tau_b, \tau_a]\,A^{(-)}$ with the same equation for B.

$$\begin{Bmatrix} T^{s}_{\frac{1}{2}\frac{1}{2}} \\ T^{s}_{-\frac{1}{2}\frac{1}{2}} \end{Bmatrix} = - \begin{Bmatrix} (m'+m) & s-\frac{1}{2}(m'^2+m^2) \\ -\frac{s+m'm}{W} & -\frac{m'+m}{2W}(s-m'm) \end{Bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} \tag{3.2}$$

with an inverse

$$\begin{Bmatrix} A \\ B \end{Bmatrix} = \frac{1}{4pp'W} \begin{Bmatrix} -\frac{m'+m}{2W}(s-m'm) & -(s-\frac{1}{2}(m'^2+m^2) \\ s+m'm & (m'+m)W \end{Bmatrix} \begin{Bmatrix} T^{s}_{\frac{1}{2}\frac{1}{2}} \\ T^{s}_{-\frac{1}{2}\frac{1}{2}} \end{Bmatrix}$$

$$\equiv N(s) \begin{Bmatrix} T^{s}_{\frac{1}{2}\frac{1}{2}} \\ T^{s}_{-\frac{1}{2}\frac{1}{2}} \end{Bmatrix} . \tag{3.3}$$

Consider the isospin odd amplitudes. They have the property that

(i) The amplitude

$$/\!\!A^{(-)} = \begin{Bmatrix} \frac{2}{s-u} A^{(-)} \\ B^{(-)} \end{Bmatrix} \tag{3.4}$$

is symmetric in $\nu = \frac{1}{2}(s-u)$

(ii) For $s \to \infty$, $t = 0$ $A^{(-)}$ is dominated by a ρ trajectory with an intercept $\alpha_\rho(0) \approx \frac{1}{2}$ and behaves like

$$/\!\!A^{(-)}_{\mathrm{F}} \sim \begin{Bmatrix} s^{\alpha_\rho - 1} \\ s^{\alpha_\rho - 1} \end{Bmatrix} . \tag{3.5}$$

(iii) For t so along the backward curve the asymptotic behaviour of A is governed by the Δ trajectory $\alpha_0 \approx 0.2$. So

$$/\!\!A^{(-)}_{\mathrm{B}} \sim \begin{Bmatrix} t^{\alpha_\Delta - \frac{3}{2}} \\ t^{\alpha_\Delta - \frac{1}{2}} \end{Bmatrix} . \tag{3.6}$$

Therefore $A^{(-)}$ satisfies the sum rule eq. (2. 3) where the summation is now over $\lambda = \frac{1}{2}, \lambda' = \frac{1}{2}, -\frac{1}{2}$ and the matrix M is

$$M(s,\theta) = \begin{pmatrix} \frac{2}{s-u} & 0 \\ 0 & 1 \end{pmatrix} N(s) . \tag{3.7}$$

Thus we get explicitly

$$A_{\mathrm{B}}^{(-)t}(0,0) = -\frac{1}{F_\pi^2} \sum \frac{1}{(\eta m_n + m')(\eta m_n + m)}$$

$$\times \begin{Bmatrix} 2\eta m_n + m' + m & -\eta(2\eta m_n + m' + m) \\ \frac{(m_n^2 + mm')(2\eta m_n + m' + m)}{\eta m_n} & -\frac{4}{m_n^2}(m_n^4 - \eta(m'+m)m_n^3 - (m+m'^2)m_n^2 \\ & -\eta m' m(m'+m)m_n - m'^2 m^2) \end{Bmatrix}$$

$$\times \begin{Bmatrix} 1 \\ \eta(-1)^{J-\frac{1}{2}}(J+\frac{1}{2}) \end{Bmatrix} \tfrac{1}{2}[G_{m'n}G_{nm} - G^*_{m'n}G^*_{nm}] . \tag{3.8}$$

In obtaining this use has been made of the reflection property (2.26) of the axial charges $X(\lambda)$ in λ.

Let us turn to the calculation of the contribution of the t-channel cut. As stated in the introduction the investigation of Höhler et al. [8] of πN backward scattering in the $I_t = 1$ state has shown, that the t-channel cut can be approximated quite well by the exchange of a single ρ meson, as far as the low t region is concerned. For the coupling strength one can take the value derived from the vector meson dominance hypothesis of the electromagnetic current. We shall assume that this situation is true for the scattering of pions on any target. For the case under consideration this gives an effective interaction

$$L_{\rho\pi\pi} = g_{\rho\pi\pi}\, \boldsymbol{\rho}^\mu \boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi} + g_{\rho\pi\pi} \frac{\kappa^V}{2m} \rho^\mu \partial^\nu \bar{\Psi}' \sigma_{\mu\nu} \frac{\tau}{2} \Psi . \tag{3.9}$$

The (unsubtracted) contribution of ρ exchange becomes then ‡

‡ The $\rho_{mm'}$ term in the lower component comes from the isovector charge contribution present for elastic transitions. For the right hand side of the equation the KSFR approximation $g_{\rho\pi\pi} \approx m_\rho / \sqrt{2} F_\pi$ has been used.

$$A\!\!\!/^{(-)} = -2\frac{g^2_{\rho\pi\pi}}{t-m^2_\rho}\begin{Bmatrix} -\dfrac{\kappa^{\rm V}}{m'+m} \\ \kappa^{\rm V}+\frac{1}{2}\delta_{mm'} \end{Bmatrix} \approx -\frac{m^2_\rho}{F^2_\pi}\,\frac{1}{t-m^2_\rho}\begin{Bmatrix} -\dfrac{\kappa^{\rm V}}{m'+m} \\ \kappa^{\rm V}+\frac{1}{2}\delta_{mm'} \end{Bmatrix} \tag{3.10}$$

At threshold this contributes

$$A'^{(-)}_{\rm B}(\mathrm{o,o}) = \frac{1}{F^2_\pi}\begin{Bmatrix} -\dfrac{\kappa^{\rm V}}{m'+m} \\ \kappa^{\rm V}+\frac{1}{2}\delta_{mm'} \end{Bmatrix} \tag{3.11}$$

Hence the resulting sum rule can be written as

$$\begin{Bmatrix} -\dfrac{(\kappa^{\rm V})m'm}{m'+m} \\ (\kappa^{\rm V})_{m'm}+\frac{1}{2}\delta_{m'm} \end{Bmatrix} = \sum_n \frac{1}{2(\eta_n m_n+m')(\eta_n m_n+m)}$$

$$\times\begin{Bmatrix} 2\eta_n m_n+m'+m & -\eta_n(2\eta_n m_n+m'+m) \\ \dfrac{(m^2_n+m'm)(2\eta_n m_n+m'+m)}{\eta_n m_n} & -\dfrac{\eta n}{m^2_n}\,[m^4_n-\eta_n(m'+m)m^3_n-(m'+m)^2 m^2_n-\eta_n \\ & \times m'm(m'+m)m_n-m'^2] \end{Bmatrix}$$

$$\times\begin{Bmatrix} 1 \\ \eta_n(-1)^{J_n-\frac{1}{2}}(J_n+\frac{1}{2}) \end{Bmatrix} (G^A_{m'n}G^A_{nm}-G^*_{m'n}G^*_{nm}) \;. \tag{3.12}$$

As a first approximation we assume that in the s-channel the sum over resonances saturates sufficiently fast to justify taking into account only the nucleon, the Roper resonance of 1470 MeV and the Δ (1238). Then our sum rule reads

$$\begin{Bmatrix} -(\kappa^{\rm V})\,m'm \\ (\kappa^{\rm V})_{m'm}+\frac{1}{2}\delta_{m'm} \end{Bmatrix} = \begin{Bmatrix} -\frac{3}{2}\dfrac{2m_\Delta+m'+m}{(m_\Delta+m')(m_\Delta+m)}G_{m'\Delta}G_{\Delta m} \\ \dfrac{m+m'}{2m}G_{m'm}G_{mm}+\dfrac{m'+m}{2m'}G_{m'm'}G_{m'm}-\mu G_{m\Delta}G_{\Delta m} \end{Bmatrix} \tag{3.13}$$

with

$$\mu = \frac{2m_\Delta^4 + \frac{1}{2}(m'+m)\,m_\Delta^3 - (m'^2 + mm' + m^2)m_\Delta^2 - \frac{1}{2}mm'(m+m')m_\Delta}{2m_\Delta^2\,(m_\Delta + m')\,(m_\Delta + m)} . \tag{3.14}$$

We now insert the explicit coupling constants of the Adler Weisberger scheme (2.29). This yields the magnetic moments

$$\left\{\begin{matrix} \kappa_{\mathrm{NN}}^{\mathrm{V}} \\ \\ \frac{1}{2} + \kappa_{\mathrm{NN}}^{\mathrm{V}} \end{matrix}\right\} = \left\{\begin{matrix} \frac{16}{7}\cos^2\theta \\ \\ 1 + 2\cos^2\theta + \frac{3}{20}\cos^4\theta \end{matrix}\right\} , \tag{3.15}$$

$$\left\{\begin{matrix} \kappa_{\mathrm{RN}}^{\mathrm{V}} \\ \\ \kappa_{\mathrm{RN}}^{\mathrm{V}} \end{matrix}\right\} = \left\{\begin{matrix} \frac{4}{3}\sin 2\theta \\ \\ (1.2 + 0.055\cos 2\theta)\sin 2\theta \end{matrix}\right\} , \tag{3.16}$$

$$\left\{\begin{matrix} \kappa_{\mathrm{RR}}^{\mathrm{V}} \\ \\ \frac{1}{2} + \kappa_{\mathrm{RR}}^{\mathrm{V}} \end{matrix}\right\} = \left\{\begin{matrix} 2.9\sin^2\theta \\ \\ 1 + 0.34\sin^2\theta - 0.31\sin^4\theta \end{matrix}\right\} . \tag{3.17}$$

As discussed in sect. 2, the angle θ should be around 45° to give a reasonable overall agreement for the coupling constants $G_{mm'}$ measured experimentally. From (3.15) we see that at $\theta = 45°$ both sum rules tend to give a somewhat too small value for $\kappa_{\mathrm{NN}}^{\mathrm{V}}$ which is experimentally $\kappa_{\mathrm{NN}}^{\mathrm{V}} = 1.85$. The best agreement is reached if one chooses $\theta = 33°$. Then we have

$$\left\{\begin{matrix} \kappa_{\mathrm{NN}}^{\mathrm{V}} \\ \\ \frac{1}{2} + \kappa_{\mathrm{NN}}^{\mathrm{V}} \end{matrix}\right\} = \left\{\begin{matrix} 1.6 \\ \\ \frac{1}{2} + 1.95 \end{matrix}\right\} . \tag{3.18}$$

The transition moment to the Roper resonance is, on the other hand, rather insensitive to the value of θ in the region around 45°. For θ between 33° and 55° on finds approximately $\kappa_{\mathrm{RN}}^{\mathrm{V}} \approx 1.2$ from both sum rules.

The elastic magnetic moment of the Roper resonance is not so reliably determined. The two sum rules (3.17) differ considerably. One has for $\theta = 33°$

$$\begin{Bmatrix} \kappa^{V}_{RR} \\ \\ \frac{1}{2} + \kappa^{V}_{RR} \end{Bmatrix} = \begin{Bmatrix} 0.9 \\ \\ \frac{1}{2} + 1.3 \end{Bmatrix} . \tag{3.19}$$

This, however, is not too surprising. In the scattering process $R\pi \leftarrow R\pi$ higher masses than that of the Roper resonance will be important in the saturation of the dispersion integral. Our result for the transition moment of the Roper resonance is the most interesting one. Experimentalists have been looking for P(1470) excitation in photo-and electroproduction, for quite some time. The data for $\pi^{o}p \leftarrow \gamma p$ and $\pi^{+}n \leftarrow \gamma p$ are by now sufficiently good to say that the positive state couples only very weakly to the photon. Since κ^{V}_{RN} is found so big we conclude that $\kappa_{R^{\circ}n} \approx -2.4$ and the resonance should be visible in the photoproduction data on neutrons *. At present only one phenomenological analysis of photoproduction has tried to accomodate a Roper coupling of the size of ours [13]. There is also some indication in recent data from Frascati [14] that Roper might be strongly coupled to neutrons. We hope that experiments will soon confirm our result.

4. THE TRANSITION MOMENT Δ Nγ

An interesting test of the strength of our method is provided by the calculation of the $\Delta N\gamma$ vertex [15]. This coupling has been analyzed quite well in recent years. Before we trust any prediction derived for other resonances it is crucial to check whether we can reproduce these coupling constants. The amplitudes free of kinematical singularities and constraints for the process

$$\Delta(p',m')\,\pi\,(q') \leftarrow N(p,m)\,\pi\,(q)$$

* Remember that the magnetic dipole amplitude is given in terms of K_{RN} at the resonance position by

$$\mathrm{Im}\, M_{1-}^{\gamma p \to \pi^{o} p} \approx -0.65\ \kappa_{R^{+}p} \sqrt{\mu b}\ , \qquad \mathrm{Im}\, M_{1-}^{\gamma u \to \pi^{o} u} \approx 0.65\ \kappa_{R^{o}n} \sqrt{\mu b}\ ,$$

and

$$\kappa^{V}_{RN} = \tfrac{1}{2}\,(\kappa_{R^{+}p} - \kappa_{R^{o}n})\ .$$

are defined by

$$\langle\pi^b(q')\,\Delta(p)|T|\,\pi^a(q')\mathrm{N}(p)\rangle = \bar{u}^\mu(p')\,[(A_1^{ba}+\not{Q}B_1^{ba})Q_\mu + (A_2^{ba}+\not{Q}B_2^{ba})K_\mu]\,u(p) \tag{4.1}$$

with

$$Q=\tfrac{1}{2}(q'+q)\;, \qquad K=\tfrac{1}{2}(q-q')\,. \tag{4.2}$$

Since Δ has isospin $\frac{3}{2}$, the isospin decomposition for this process is given by

$$A^{ba} = A^{(-)}I_-^{ba} + A^{(+)}I_+^{ba} \tag{4.3}$$

where

$$I_\pm^{ba,c} = \tfrac{1}{2}\,(\delta^{cb}\,\tau^a \pm \delta^{ca}\,\tau^b)\,. \tag{4.4}$$

In terms of A and B the s-channel helicity amplitudes * are given by

$$\begin{pmatrix} \hat{T}_{\frac{3}{2},\frac{1}{2}} \\ \sqrt{3}\,\hat{T}_{\frac{1}{2},\frac{1}{2}} \\ \sqrt{3}\,\hat{T}_{-\frac{1}{2},\frac{1}{2}} \\ \hat{T}_{-\frac{3}{2},\frac{1}{2}} \end{pmatrix} = N^{-1} \begin{pmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \end{pmatrix} \tag{4.5}$$

where

$$N^{-1} = \begin{pmatrix} pS_- & -pS_+\cos^2\frac{1}{2}\theta + K_1S_- & -pS_-\sin^2\frac{1}{2}\theta - K_1S_+ & pS_+ \\ pS_- & -pS_+\cos^2\frac{1}{2}\theta + K_2S_- & -pS_-\sin^2\frac{1}{2}\theta - K_2S_+ & pS_+ \\ -pS_+ & pS_-\cos^2\frac{1}{2}\theta - K_1S_+ & pS_+\sin^2\frac{1}{2}\theta + K_1S_- & -pS_- \\ -pS_+ & pS_-\cos^2\frac{1}{2}\theta - K_2S_+ & pS_+\sin^2\frac{1}{2}\theta + K_2S_- & -pS_- \end{pmatrix} \times$$

* The nucleon is particle 1, Jacob-Wick conventions are used.

$$\times \begin{pmatrix} 1 & 0 & \frac{1}{2}(m'-m) & 0 \\ 0 & 1 & 0 & \frac{1}{2}(m'-m) \\ 0 & 0 & W & 0 \\ 0 & 0 & 0 & W \end{pmatrix} . \tag{4.6}$$

Here $S_{\pm} = \sinh \frac{1}{2}(\xi \pm \xi')$ and

$$K_1 = \frac{1}{M}(2p'\omega + p'E + E'p \cos\theta), \qquad K_2 = \frac{1}{M}(-Ep' + E'p\cos\theta) .$$

and ζ and ζ' denote the rapidities of nucleon and Δ respectively. Inverting these e-quations we find

$$N = -\frac{m'^2 m}{\sqrt{2}p^2p'^2 W} \begin{pmatrix} 1 & 0 & -\frac{m'-m}{2W} & 0 \\ 0 & 1 & 0 & -\frac{m'-m}{2W} \\ 0 & 0 & \frac{1}{W} & 0 \\ 0 & 0 & 0 & \frac{1}{W} \end{pmatrix}$$

$$\times \begin{pmatrix} p\cos^2\frac{1}{2}\theta\, S_+ - K_2 S_- & pS_- & pS_+ & p\sin^2\frac{1}{2}\theta\, S_- + K_2 S_+ \\ -p\cos^2\frac{1}{2}\theta\, S_+ + K_1 S_- & -pS_- & -pS_+ & -pS_+ - p\sin^2\frac{1}{2}\theta\, S_- - K_1 S_+ \\ p\cos^2\frac{1}{2}\theta\, S_- - K_2 S_+ & pS_+ & pS_- & pS\_p\sin^2\frac{1}{2}\theta\, S_+ + K_2 S_- \\ -p\cos^2\frac{1}{2}\theta\, S_- + K_1 S_+ & -pS_+ & -pS_- & -p\sin^2\frac{1}{2}\theta\, S_+ - K_1 S_- \end{pmatrix} \tag{4.7}$$

Since the pion mass is taken to be zero, we can write

$$\sinh\tfrac{1}{2}(\zeta+\zeta') = \frac{1}{2\sqrt{m'm}}\,\frac{s-m'm}{W}, \qquad \sinh\tfrac{1}{2}(\zeta-\zeta') = \frac{m'-m}{2\sqrt{mm}},$$

$$\cosh\tfrac{1}{2}(\zeta+\zeta') = \frac{1}{2\sqrt{m'm}}\,\frac{s+m'm}{W}, \qquad \cosh\tfrac{1}{2}(\zeta-\zeta') = \frac{m'+m}{2\sqrt{m'm}}. \tag{4.8}$$

One obtains then

$$N = -\frac{\sqrt{m'm}}{8\sqrt{2p^2p'^2W^3}}\left(N_{\mathrm{F}}\cos^2\frac{\theta}{2} + N_{\mathrm{B}}\sin^2\frac{\theta}{2}\right) \tag{4.9}$$

where N_{F} and N_{B} are given, respectively, by

$$N_{\mathrm{F}} = \begin{pmatrix} m'S_{12}(S_{12}+S_{34}) - M_-^2 M_+\tilde{\varphi} & \frac{m'}{W}S_{12}M_-\tilde{\varphi} & m'S_{12}(S_{12}+S_{34}) & WM_+M_-(S_{12}+S_{34}) \\ -mS_{34}(S_{12}+S_{34}) & -\frac{m'}{W}S_{12}M_-\tilde{\varphi} & -m'S_{12}(S_{12}+S_{34}) & -W(S_{12}+S_{34})^2 \\ -2mS_{34}M_- & 2\frac{m'}{W}S_{12}\tilde{\psi} & 2m'S_{12}M_- & 2WM_+^2M_- \\ 2S_{34}(\tilde{\psi}+S_{12}) & -2\frac{m'}{W}S_{12}\tilde{\psi} & -2m'S_{12}M_- & -2WM_-(S_{12}+S_{34}) \end{pmatrix} \tag{4.10}$$

and

$$N_{\mathrm{B}} = \begin{pmatrix} \frac{M'_-}{s}\tilde{\varphi}^2\tilde{\psi} & \frac{m'}{W}S_{12}M_-\tilde{\varphi} & S_{12}m'(S_{12}+S_{34}) & -\frac{1}{W}S_{34}\tilde{\varphi}(S_{12}+\tilde{\psi}) \\ -\frac{M_-}{s}\tilde{\varphi}(2_sS_{34}-\tilde{\psi}\tilde{\psi}) & -\frac{m'}{W}S_{12}M_-\tilde{\varphi} & -S_{12}m'(S_{12}+S_{34}) & -\frac{1}{W}S_{34}(2_sS_{34}-mM_+\tilde{\psi}) \\ \frac{2}{s}\tilde{\varphi}\tilde{\psi}^2 & \frac{2m'}{W}S_{12}\tilde{\psi} & 2m'S_{12}M_- & \frac{2}{W}m\tilde{\psi}S_{34} \\ \frac{2}{s}\tilde{\psi}(2_sS_{34}-\tilde{\psi}\tilde{\psi}). & -\frac{2m'}{W}S_{12}\tilde{\psi} & -2m'S_{12}M_- & -\frac{2}{W}S_{34}(m\tilde{\psi}+2_sM_-) \end{pmatrix} \tag{4.11}$$

with

$\tilde{\varphi} = (s + m'm)$, $\tilde{\psi} = (s - m'm)$, $S_{12} = (s - m^2)$, $S_{34} = (s - m'^2)$, $M_\pm = (m' \pm m)$.

Consider the isospin odd amplitudes

$$A^{(-)} = \begin{pmatrix} A_1^{(-)} \\ \frac{2}{s-u} A_2^{(-)} \\ \frac{2}{s-u} B_1^{(-)} \\ B_2^{(-)} \end{pmatrix} . \tag{4.12}$$

They are all even functions in ν and behave as

$$A^{(-)} \sim \begin{pmatrix} s^{\alpha_\rho - 1} \\ s^{\alpha_\rho - 1} \\ s^{\alpha_\rho - 3} \\ s^{\alpha_\rho - 1} \end{pmatrix} , \qquad \alpha_\rho \approx 0.5 , \tag{4.13}$$

for large s in forward scattering and as $t^{\alpha_\Delta - \frac{1}{2}} \approx t^{-0.3}$ for large t in the backward direction. Thus the sum rules (2.23) hold again with

$$M(s,t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{2}{s-u} & 0 & 0 \\ 0 & 0 & \frac{2}{s-u} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} N(s,\theta_s) . \tag{4.14}$$

The evaluation of the sum of the baryon contributions is a cumbersome numerical calculation from which not much physical insight can be gained. In order to get a first estimate we take an SU(4) type approximation in which N and Δ are the only baryon resonances in the s-channel with equal mass. Then the matrix M simplifies considerably. We find in forward direction

$$M(s,\mathrm{o}) = -\frac{2\sqrt{2}mW}{(s-m^2)^2}\begin{pmatrix} m & 0 & m & 0 \\ -m & 0 & -m & -2W \\ 0 & \frac{m}{W} & 0 & 0 \\ 2 & -\frac{m}{W} & 0 & 0 \end{pmatrix}, \tag{4.15}$$

while at $\theta_s = \pi$ ($t = t_n$)

$$M(s,t_n) = -\frac{2\sqrt{2}mW}{(s-m^2)^2}\begin{pmatrix} 0 & 0 & m & -\frac{s+m^2}{W} \\ 0 & 0 & -m & -\frac{s+m^2}{W} \\ \frac{s+m^2}{s} & \frac{m}{W} & 0 & -\frac{m}{W} \\ \frac{s-m^2}{s} & \frac{m}{W} & 0 & -\frac{m}{W} \end{pmatrix}. \tag{4.16}$$

If one uses again the reflection properties (2.26) of X one finds for the contribution of N and Δ

$$[A_{\mathrm{B}}^{t(-)}(\mathrm{o},\mathrm{o})] = \frac{1}{F_\pi^2}\frac{\sqrt{6}}{2m}\left\{\begin{pmatrix} -m \\ 0 \\ 0 \\ 1 \end{pmatrix}[X_b(\tfrac{1}{2}), X_a(\tfrac{1}{2})]_{\mathrm{N}} + \begin{pmatrix} 5m \\ \frac{3}{2}m \\ -\frac{3}{2}m \\ 5 \end{pmatrix}[X_b(\tfrac{1}{2}), X_a(\tfrac{1}{2})]_{\Delta}\right\}, \tag{4.17}$$

and going to the reduced matrix elements G we obtain

$$[A_{\mathrm{B}}^{t(-)}(\mathrm{o},\mathrm{o})] = \frac{1}{F_\pi^2}\,\frac{3}{2\sqrt{2}m}\begin{Bmatrix} -m\,(G^* G_{\mathrm{A}} + 25 G_{\mathrm{A}}^* G^*) \\ -\frac{1}{m}\,\frac{15}{2} G_{\mathrm{A}}^* G^* \\ \frac{1}{m}\,\frac{15}{2} G_{\mathrm{A}}^* G^* \\ -G^* G_{\mathrm{A}} + 25 G_{\mathrm{A}}^* G^* \end{Bmatrix} \tag{4.18}$$

Now consider the exchange of a ρ meson in the t-channel. With the vector meson dominance hypothesis the coupling of the ρ current to N and Δ is given by

$$\begin{aligned}\langle \Delta(p'),\lambda' | j^{\mu}_{\rho a} | \mathrm{N}(p),\lambda\rangle = g_{\rho\pi\pi}\bar{u}^{\nu}(\lambda',p')\,[C_3\,(\not{q}\, g^{\mu}_{\nu} - q_{\nu}\gamma^{\mu}) \\ + C_4\,((qp')g^{\mu}_{\nu} - q_{\nu}p'^{\mu}) \\ + C_5\,((qp)g^{\mu}_{\nu})]\; u(p,\lambda) \\ \times \sqrt{\tfrac{3}{2}}\,\chi_a^{+}\chi\end{aligned} \tag{4.19}$$

where C_3, C_4, C_5 are the electromagnetic coupling constants as defined by Gourdin and Salin [17] and χ_a and χ_b are the isospinors of Δ and N, respectively. With this coupling the contribution of ρ to $A^{(-)}$ for degenerate Δ and N masses is

$$[A_{\mathrm{B}}^{t(-)}(\mathrm{o},\mathrm{o})] = \frac{\sqrt{6}}{F_\pi^2}\,\frac{m_\rho^2}{t-m_\rho^2}\begin{Bmatrix} -2m\,C_3 + \frac{1}{2}t\,(C_4 - C_5) \\ -(C_4 + C_5) \\ 0 \\ -2C_3 \end{Bmatrix}\,. \tag{4.20}$$

Therefore the unsubtracted contribution at threshold is for degenerate Δ and N masses:

$$[A_{\mathrm{B}}^{t(-)}(\mathrm{o},\mathrm{o})] = \frac{\sqrt{6}}{F_\pi^2}\begin{Bmatrix} -2m\,C_3 + \frac{m_\rho^2}{2}(C_4 - C_5) \\ -(C_4 + C_5) \\ 0 \\ -2C_3 \end{Bmatrix} . \tag{4.21}$$

We insert this result in equation (4.18) and use for the coupling constants G the SU(4) numbers

$$(G_{\mathrm{A}}, G^*, G_{\mathrm{A}}^*) = (\tfrac{5}{3}, \tfrac{4}{3}, \tfrac{1}{3}) . \tag{4.22}$$

The result is

$$C_3 m = \frac{5}{\sqrt{3}} , \qquad C_4 m^2 = C_5 m^2 = \tfrac{1}{4}\,\frac{5}{\sqrt{3}} . \tag{4.23}$$

The experimental situation concerning these couplings constants is the following: The magnetic and electric decay widths of $\Delta^+ \to p\gamma$ are given by ‡

$$\Gamma_{\mathrm{m}_1} = \frac{e^2}{4\pi}\,\frac{(M^2-m^2)^3}{(2M)^3}\,\frac{1}{12M^2}\,|\,(3M+m)\,C_3 - M(M-m)\,(C_4 + C_5)\,|^2 \tag{4.24}$$

$$\Gamma_{\mathrm{e}_2} = \frac{e^2}{4\pi}\,\frac{(M^2-m^2)^3}{(2M)^3}\,\frac{1}{4M^2}\,|\,(M-m)\,C_3 - M(M-m)\,(C_4 + C_5)\,|^2 \tag{4.25}$$

‡ The magnetic moment is normally [18] defined as

$$\mu^{*2} = \frac{2mm'}{\alpha k^{*3}}\,\Gamma_{\mathrm{m}_1} \approx 17.8 \times 10.^3\,\mathrm{GeV}^{-1}\,\Gamma_{\mathrm{m}_1} .$$

Experimentally $\Gamma_{\mathrm{m}_1} \approx 69.10^{-3}$ BeV and therefore $\mu^* = 3.5$ as found by all analyses of photo and electroproduction data. Note that also the measurement of Ash et al. [21] gives $\mu^* = 3.45$ even though the authors claim to have measured the value of $\mu^* = 3$. The reason is that they have mistaken $\sqrt{m/m'}\,\mu^* \approx 0.87\,\mu^*$ for the magnetic moment μ^* as first defined by Dalitz and Sutherland [18]. Thus their value of $\sqrt{m/m'}\,\mu^* \approx 3$ is in perfect agreement with the result of electroproduction data (The literature has been rather confused about this point. See for example the review article of Pfeil and Schwela [19] (p.225) and ref. [20] (p.282)). The correct normalization of the form factors (of all resonances) is given in ref. [22].

The analysis of photoproduction data $\pi^0 p \leftarrow \gamma p$ is usually done in terms of the amplitudes M_{1+} and E_{1+} of CGLN [23]
They are connected with Γ_{m_1} and Γ_{e_2} by

$$\frac{\Gamma_{m_1} F_{\pi op}}{\Gamma^2_{tot}} = K^* q^* \, 2 |M_{1+}|^2 \ , \tag{4.26}$$

$$\frac{\Gamma_{e_2} \Gamma_{\pi op}}{\Gamma^2_{tot}} = K^* q^* 6 |E_{1+}|^2 \ , \tag{4.27}$$

where k^* and q^* are the c.m. momenta of γ and π^0 respectively. If one takes propes care of the sign one obtains for the ratio

$$\frac{M_{1+}}{E_{1+}} = -\frac{(3m'+m) - m'(m'-m)(C_4+C_5)/C_3}{m'-m-m'(m'-m)(C_4+C_5)/C_3} \approx \frac{15.5 - 1.32\,x}{1-1.32\,x} \ . \tag{4.28}$$

Using the experimental values of $\Gamma_\gamma = 0.69 \times 10^{-3}$ BeV [19] or Im $M^{\gamma p \to \pi^0 p}_{1+}$ $\approx 3.28 \sqrt{\mu b} = 0.167$ BeV^{-1} [12, 19] together with the ratio $E_{1+}/M_{1+} \approx -0.05$ [15] we find

$$C_3 m_p \approx 2.0 \ , \qquad x \approx 0.16 \ .$$

Our result, on the other hand, implies

$$C_3 m_p \approx \frac{5}{\sqrt{3}} \frac{mp}{m} \approx 2.5 \ , \qquad x \approx \tfrac{1}{2} \frac{mp}{m} \approx .4 \tag{4.29}$$

To be consistent with our approximation we have inserted for m the average mass of nucleon and Δ resonance. We see that the agreement is quite reasonable considering the roughness of the approximation involved.

5. CONCLUSION

Our program of

(i) Equating forward and backward dispersion relations at threshold

(ii) Using for the baryon contributions in the s-channel sharp resonances with coupling constants taken from algebraic saturation schemes of Adler Weisberger relations

(iii) Saturating the t-channel dispersion integral of $I_t = 1$ amplitudes with a single ρ meson

(iv) Relating the coupling constants of the ρ meson to the electromagnetic coupling via the vector meson dominance hypothesis appears to provide a simple way of determining electromagnetic couplings of higher resonances. It has been tested to work in the calculation of the anomalous magnetic moment of the nucleons and the transition $\Delta N\gamma$. As a first prediction we have obtained a large isovector coupling for the Roper nucleon transition. The evaluation of transition moments of higher resonances will be done in the future. As a final remark we want to mention that the same methods can also be applied to other scattering processes as photoproduction of pions and compton scattering on nucleons in order to obtain information on the coupling strengths of mesons exchanged in the t-channel. We refer the reader to the literature for further reference [24, 25].

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