

# New Gauge Symmetry in Gravity and the Evanescent Role of Torsion

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**Abstract:** If the Einstein-Hilbert action  $\mathcal{L}_{\text{EH}} \propto R$  is re-expressed in Riemann-Cartan spacetime using the gauge fields of translations, the vierbein field  $h^\alpha_\mu$ , and the gauge field of local Lorentz transformations, the spin connection  $A_{\mu\alpha}{}^\beta$ , there exists a new gauge symmetry which permits reshuffling the torsion, partially or totally, into the Cartan curvature term of the Einstein tensor, and back, via a *new multivalued gauge transformation*. Torsion can be chosen at will by an arbitrary gauge fixing functional. There exist many equivalent ways of specifying the theory, for instance Einstein's traditional way where  $\mathcal{L}_{\text{EH}}$  is expressed completely in terms of the metric  $g_{\mu\nu} = h^\alpha_\mu h_{\alpha\nu}$ , and the torsion is zero, or Einstein's teleparallel formulation, where  $\mathcal{L}_{\text{EH}}$  is expressed in terms of the torsion tensor, or an infinity of intermediate ways. As far as the gravitational field in the far-zone of a celestial object is concerned, matter composed of spinning particles can be replaced by matter with only orbital angular momentum, without changing the long-distance forces, no matter which of the various new gauge representations is used.

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1. In theoretical physics it often happens that a mathematical structure has a simple extension for which a natural phenomenon is waiting to be discovered. The most prominent example is the existence of a negative square root of the relativistic mass shell relation  $p_0 = \sqrt{\mathbf{p}^2 + m^2}$  which led Dirac to postulate the existence of a positron, discovered in 1932 by Carl Anderson [1]. Sometimes, this rule does not seem to work initially, only to find out later that nature has chosen an unexpected way to make it work after all. Here the best example is the existence of a solution of the above energy-momentum

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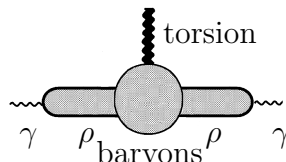
relation for negative  $m^2$ , which was interpreted by some theoreticians as the signal for the existence of a particle faster than light. Such particles were never found. A simple physical realization appeared, however, with the discovery of the Ginzburg-Landau field theory of phase transitions and its quantum versions (now referred to as Higgs field theory). Since there are always *interactions*, a negative parameter  $m^2$  destabilizes the field fluctuations. The fields move to a new ground state, around which they fluctuate with *positive*  $m^2$ . The situation is completely analogous to what happens in any building if  $\omega^2$  of one of its eigenfrequencies turns negative. The building collapses until the debris settles in a ruin, and that has only positive  $\omega^2$ 's. The collapse of an interacting field system with negative  $m^2$  is observed as a phase transition to a state with stable fluctuations and positive  $m^2$ .

2. For many years, theorists have been wondering why Einstein's theory of gravity represents such a perfect geometrization of the gravitational forces [2]. Since the work of Cartan in 1922 it is known that the Riemannian spacetime, in which the celestial objects move, has a "natural" extension to *Riemann-Cartan* spacetime. This possesses a further geometric property called *torsion*. Why is there no trace of it in the movements of planets? Einstein himself has asked this question and discussed it in letters with Cartan [3]. He set up a theory of *teleparallelism* which explains gravity by a theory in Riemann-Cartan spacetime, in which the total curvature tensor vanishes identically. The Einstein-Hilbert action is then equal to a combination of scalars formed from torsion tensors [4], and torsion forces provide us merely with an alternative way of describing gravitational forces, as emphasized in Refs. [5, 6].

3. Yet another extension of Einstein's theory to Riemann-Cartan spacetime was advanced since 1959 [7, 8, 9]. It has the appealing feature that it can be rewritten as a gauge theory invariant under local Poincaré transformations, i.e., both local translations and local Lorentz transformations, thus bringing it to a similar form as the gauge theories of weak, electromagnetic, and strong interactions. This gauge theory treats torsion as an *independent* field which couples only to the intrinsic spin of the elementary particles in a celestial body. Unfortunately, however, such an approach has several unsatisfactory features. First, the theory is meant to be classical, but the spin carries a power of  $\hbar$  which vanishes in the classical limit. So there is really no classical source of torsion. Indeed, if torsion couples to spin with the gravitational coupling strength, the factor  $\hbar$  implies that it cannot play any sizable role in the forces between celestial bodies. For example, even if the earth consisted only of polarized atoms, its intrinsic spin would be  $10^{-15}$  times smaller than the rotational spin around the axis.

Moreover, there exist severe conceptual problems. One was emphasized in Ref. [10]. As long as we do not know precisely the truly *elementary particles*, and it is doubtful that we ever will, many particles are described by effective fields, and it is impossible to specify whether the spin of those fields is caused by orbital motion or by the intrinsic spins of more elementary constituents. As an example, the spin-one field of a  $\rho$ -meson contains a wave function of two spinless pions in a p-wave, which do not couple to torsion. But it also contains two spin- $\frac{1}{2}$  quarks in an s-wave which would couple. Another problem

is that if torsion couples to all spins, the photon becomes massive. In order to avoid this, the authors advocating this approach postulate that the photon is an exception, and is not coupled to torsion. However, this contradicts the fact that roughly one percent of a photon is a virtual  $\rho$ -meson, which is strongly coupled to baryons. These, in turn, are supposed to be coupled to torsion (see Fig. 1), so the photon would become massive after all. Thus the existence of an independent torsion field is highly dubious, and we may

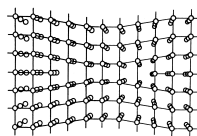


**Figure 1** Diagram for mass generation of photon. It couples via a  $\rho$ -meson to baryonic matter which would be coupled to torsion if  $q \neq 1$ .

ask ourselves, whether the description of gravity in Riemann-Cartan spacetime proposed in Refs. [7, 8, 9] has really a chance of being true, or whether nature doesn't have a deeper reason for avoiding the above problems. It is the purpose of this note to answer this question affirmatively. Inspiration comes from a simple model of gravity, a “world crystal” with defects [11, 9, 12], whose lattice constant is of the order of a Planck length. Some consequences of such a world crystal were pointed out in a recent study of black holes in such a scenario [13].

4. We begin by showing that in the absence of matter, a world crystal is a model for Einstein's theory with a new type of extra gauge symmetry in which zero torsion is merely a special gauge. A completely equivalent gauge is the absence of Cartan curvature, which is found in Einstein's teleparallel universe. Before presenting the argument, recall that a crystal can have two different types of topological line-like defects [8, 9], which in a four-dimensional world crystal are world surfaces (which may be the objects of string theory).

First, there are translational defects called *dislocations* (Fig. 2). These are produced

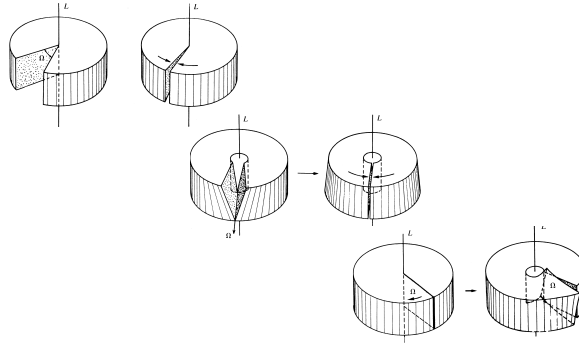


**Figure 2** Formation of a dislocation line (of the edge type) by a Volterra process. The Burgers vector  $\mathbf{b}$  characterizes the missing layer. There exist two more types where  $\mathbf{b}$  points in orthogonal directions.

by a cutting process due to Volterra: a single-atom layer is removed from the crystal, allowing the remaining atoms to relax to equilibrium under the elastic forces. A second type of topological defects is of the rotation type, the so-called *disclinations* (Fig. 3). They arise by removing an entire wedge from the crystal and re-gluing the free surfaces. The defects imply a failure of derivatives to commute in front of the displacement field  $u_i(\mathbf{x})$ . In three dimensions, the dislocation density is given by the tensor

$$\alpha_{ij}(\mathbf{x}) = \epsilon_{ikl} \nabla_k \nabla_l u_j(\mathbf{x}). \quad (1)$$

If  $\omega_i \equiv \frac{1}{2} \epsilon_{ijk} [\nabla_j u_k(\mathbf{x}) - \nabla_k u_j(\mathbf{x})]$  denotes the local rotation field, the disclination density



**Figure 3** Three different possibilities of constructing disclinations: wedge, splay, and twist disclinations. They are characterized by the Frank vector  $\Omega$ .

is defined by

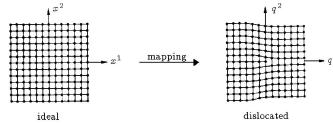
$$\theta_{ij}(\mathbf{x}) = \epsilon_{ikl} \nabla_k \nabla_l \omega_j(\mathbf{x}). \quad (2)$$

The defect densities satisfy the conservation laws

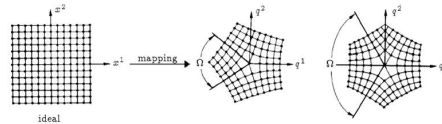
$$\nabla_i \theta_{ij} = 0, \quad \nabla_i \alpha_{ij} = -\epsilon_{jkl} \theta_{kl}. \quad (3)$$

These are fulfilled as Bianchi identities if we express  $\theta_{ij}(\mathbf{x})$ ,  $\alpha_{ij}(\mathbf{x})$  in terms of plastic gauge fields  $\beta_{kl}^p$ ,  $\phi_{lj}^p$ , setting  $\theta_{ij} = \epsilon_{ikl} \nabla_k \phi_{lj}^p$ ,  $\alpha_{il} = \epsilon_{ijk} \nabla_j \beta_{kl}^p + \delta_{il} \phi_{kk}^p - \phi_{li}^p$ . The defect densities are invariant under the gauge transformations  $\beta_{kl}^p \rightarrow \beta_{kl}^p + \nabla_k u_l^p - \epsilon_{klr} \omega_r^p$ ,  $\phi_{li}^p \rightarrow \phi_{li}^p + \partial_l u_i^p$ , where  $\omega_i^p \equiv \frac{1}{2} \epsilon_{ijk} \nabla_j u_k^p$ . Thus  $h_{ij} \equiv \beta_{ij}^p + \epsilon_{ijk} \omega_k^p$  and  $A_{ijk} \equiv \phi_{ij}^p \epsilon_{jkl}$  are *translational* and *rotational* defect gauge fields in the crystal [14].

The Volterra processes can be represented mathematically by multivalued transformations from an Euclidean crystal with coordinates  $\bar{x}^a$  to a crystal with defects and coordinates  $x^\mu$ , as illustrated in Figs. 4 and 5 for two-dimensional crystals. For an edge dislo-



**Figure 4** Multivalued mapping of the perfect crystal to an edge dislocation with a Burgers vector  $\mathbf{b}$  pointing in the 2-direction.



**Figure 5** Multivalued mapping of the perfect crystal to a wedge disclination of Frank vector  $\Omega$  in the third direction.

cation the mapping is  $\bar{x}^1 = x^1$ ,  $\bar{x}^2 = x^2 + (b/2\pi)\phi(x)$ , where  $\phi(x) \equiv (1/2\pi) \arctan(x^2/x^1)$ . Initially, this function has a cut from the origin towards left infinity. In a second step, the cut is removed and the multivalued version of the arctan is taken. This makes  $\phi(\mathbf{x})$  the Green function of the commutator  $[\partial_1, \partial_2]$ :  $(\partial_1 \partial_2 - \partial_2 \partial_1)\phi(x) = 2\pi \delta^{(2)}(\mathbf{x})$ . For a wedge disclination, the mapping is  $d\bar{x}^i = \delta^i_\mu [x^\mu + (\Omega/2\pi)\epsilon^\mu_\nu x^\nu \phi(x)]$ .

A combination of the two

$$\eta_{ij}(\mathbf{x}) \equiv \theta_{ij}(\mathbf{x}) - \frac{1}{2} \nabla_m [\epsilon_{min} \alpha_{jn}(\mathbf{x}) + \{ij\} + \epsilon_{ijn} \alpha_{mn}] \quad (4)$$

forms the *defect tensor*

$$\eta_{ij}(\mathbf{x}) \equiv \epsilon_{ikl}\epsilon_{jmn}\nabla_k\nabla_m u_{ln}^p(\mathbf{x}), \quad u_{ln}^p \equiv \frac{1}{2}(\beta_{ln}^p + \beta_{nl}^p). \quad (5)$$

It is a symmetric tensor due to the conservation laws (3), and represents the Einstein tensor  $G_{ij} \equiv R_{ij} - \frac{1}{2}g_{ij}R_k^k$  of the geometry of the world crystal.

The expressions can easily be defined on a simple-cubic world lattice if we replace  $\nabla_i$  by lattice derivatives, as shown in [8, 9]. There it is also shown that, in three spacetime dimensions, the disclination density  $\theta_{ij}(\mathbf{x})$  represents the Einstein tensor  $G_{ij}^C$  associated with the *Cartan curvature tensor*  $R_{ijk}^C$  of the Riemann-Cartan geometry of the world crystal. The relation is

$$G_{ji}^C(\mathbf{x}) = \epsilon_{ikl}\nabla_k\nabla_l\omega_j(\mathbf{x}) = \theta_{ij}(\mathbf{x}). \quad (6)$$

The dislocation density  $\alpha_{ij}(\mathbf{x})$  represents the torsion  $S_{lkj} = \frac{1}{2}(\Gamma_{lkj} - \Gamma_{klj})$  of the Riemann-Cartan geometry. Here the relation is

$$\alpha_{ij} = \epsilon_{ikl}S_{lkj}. \quad (7)$$

**5.** The standard form of a defect with Burgers vector  $b_l$  and Frank vector  $\Omega_q$  has a displacement field

$$u_l(\mathbf{x}) = -\delta(\mathbf{x}; V)[b_l + \epsilon_{lqr}\Omega_q(x_r - \bar{x}_r)], \quad (8)$$

where  $\epsilon_{lqr}$  is the antisymmetric unit tensor,  $\bar{x}_r$  the axis of rotation of the disclination part, and  $\delta(\mathbf{x}; V)$  is the delta function on the volume  $V$ , i.e., in three dimensions:

$$\delta(\mathbf{x}; V) = \int_V d^3x' \delta^{(3)}(\mathbf{x} - \mathbf{x}'). \quad (9)$$

Its derivative is the delta function on the Volterra surface  $S$  of  $V$ :

$$-\nabla\delta(\mathbf{x}; V) = \delta(\mathbf{x}; S) = \int_S d\mathbf{S}' \delta^{(3)}(\mathbf{x} - \mathbf{x}'). \quad (10)$$

For the new gauge symmetry, the crucial observation is that as a simple consequence of (8), a dislocation line in the world crystal can either be obtained by a Volterra process of cutting out a thin slice of material of thickness  $\mathbf{b}$ , or alternatively by cutting out a wedge of Frank vector  $\mathbf{\Omega}$ , and reinserting it at distance  $\mathbf{b}$  from the cut. Thus the dislocation line is indistinguishable from a pair of disclination lines with opposite Frank vector  $\mathbf{\Omega}$  whose axes of rotation are separated by a distance  $\mathbf{b}$  (Fig. 6a). Conversely, a disclination line is equivalent to a stack of dislocation lines with fixed Burgers vector  $\mathbf{b}$  (Fig. 6b).



**Figure 6** Equivalence between a) dislocation and pair of disclination lines, b) disclination and stack of dislocation lines.

Analytically, this is most easily seen in the two-dimensional version of the relation (4):

$$\eta_{33} = \theta_{33} + \epsilon_{3mn} \nabla_m \alpha_{3n}. \quad (11)$$

Each term is invariant under the plastic gauge transformations  $\beta_{kl}^p \rightarrow \beta_{kl}^p + \nabla_k u_l^p - \epsilon_{kl} \omega_3^p$ ,  $\phi_l^p \rightarrow \phi_l^p + \partial_l \omega_3^p$ . The general defect has a displacement field

$$u_l = -\delta(V_2)[b_l - \Omega \epsilon_{3lr}(x_r - \bar{x}_r)]. \quad (12)$$

The first term is a dislocation, the second term a disclination. According to Fig. 6, the latter can be read as a superposition of dislocations with the same Burgers vector  $\tilde{b}_l = -\int_{\bar{x}}^x dx'_r \Omega \epsilon_{3lr}$ . The former may be viewed as a dipole of disclinations:  $-\bar{\nabla}_l[-\frac{1}{2}b_m \epsilon_{3km}] \epsilon_{3kr}(x_r - \bar{x}_r)$ .

**6.** Let us now derive the emerging gravitational forces in the world crystal. Consider the partition function, at unit temperature, of the world crystal which we take to be three-dimensional, for simplicity:

$$Z = \sum_{n_{ij}(\mathbf{x})} \prod_{\mathbf{x}, i} \left[ \frac{du_i(\mathbf{x})}{a} \right] e^{-H}. \quad (13)$$

In linear elasticity, the energy depends quadratically only on the difference between the elastic and the plastic strain tensors  $u_{ij} = \frac{1}{2}(\nabla_i u_j + \nabla_j u_i)$  and  $u_{ij}^p$ , and reads on the lattice

$$H = \mu \sum_{\mathbf{x}} \sum_{i,j} [\nabla_i u_j(\mathbf{x}) + \nabla_j u_i(\mathbf{x}) - n_{ij}(\mathbf{x})]^2. \quad (14)$$

Here  $\mu$  is the elastic constant [16], and the integer numbers  $n_{ij}$  of are the lattice versions of  $2u_{ij}^p$  in Eq. (5). This partition function explains for low temperature the correct classical specific heat. If the temperature is increased, it reaches a point where the configuration entropy of the defects wins over the Boltzmann factors of their energy, and the world crystal melts.

We have shown in [9] that in order to arrive at the proper Newton forces at long distances we have to insert one more derivative in the lattice action and start out with what is called the *floppy world crystal* where

$$H = \mu \sum_{\mathbf{x}} \sum_{i,j} \{ \nabla_k [\nabla_i u_j(\mathbf{x}) + \nabla_j u_i(\mathbf{x}) - n_{ij}(\mathbf{x})] \}^2. \quad (15)$$

The partition function depends on the defect configuration only via the defect tensor formed from  $n_{ij}$ , i.e., on  $\eta_{ij}$ . It is a functional of this tensor which can be expanded into powers of  $\eta_i^i$ ,  $\eta_{ij}\eta^{ij}$ ,  $\eta_{ij}\eta^{jk}\eta_k^i$ ,  $\eta_{ij}\eta^{ij}\eta_k^k$ ,  $\dots$ . The expansion coefficient are proportional to powers of the Planck length, for each  $\eta_{ij}$  two powers. Since  $\eta_{ij}$  is the defect representation of the Einstein tensor  $G_{ij}$ , the partition function defines a gravitational action which is a power series of the Einstein tensor. To leading (second) order in the Planck length is proportional to the scalar  $G = G_i^i = -R$ .

Note that the gravitational action arises in this model from the *entropy* of the fluctuations [12] in the same way as rubber elasticity in polymer physics [17].

The defect tensor  $\eta_{ij} \hat{=} G_{ij}$  can be decomposed into an Cartan part and a torsion part as in Eq. (4). From the equivalence of defects illustrated in Fig. 6 it is now obvious that we can re-express the action, which contains only to defect tensor  $\eta_{ij} \hat{=} G_{ij}$ , completely in terms of the dislocation density  $\alpha_{ij}$ , i.e., in terms of the torsion tensor  $S_{lkj}$  via Eq. (7). The Cartan curvature tensor is then identically zero, showing that Einstein's teleparallel formulation of gravity is completely equivalent to the original Einstein theory. Alternatively, we may make the torsion vanish identically, and recover the original Einstein theory.

In addition, there exists an infinite number of intermediate formulations of the theory with both Riemann-Cartan curvature and torsion in some well-defined mixture.

**7.** Generalizing the defect relations (4) and (11) to  $D \geq 4$  spacetime dimensions and allowing for large deviations from Euclidean space, we find [18]

$$G_{\mu\nu} = G_{\mu\nu}^C - \frac{1}{2} D^{*\lambda} (S_{\mu\nu,\lambda} - S_{\nu\lambda,\mu} + S_{\lambda\mu,\nu}) \quad (16)$$

where  $G_{\mu\nu}$  is the Einstein tensor and  $G_{\mu\nu}^C$  its Cartan version, while  $S_{\mu\kappa}{}^\tau$  is the Palatini tensor related to the torsion field  $S_{\mu\kappa}{}^\tau$  by

$$\frac{1}{2} S_{\mu\kappa}{}^\tau \equiv S_{\mu\kappa}{}^\tau + \delta_\mu{}^\tau S_{\kappa\lambda}{}^\lambda - \delta_\kappa{}^\tau S_{\mu\lambda}{}^\lambda. \quad (17)$$

The symbol  $D_\mu^*$  denotes the covariant derivative  $D_\mu^* \equiv D_\mu^C + 2S_{\mu\kappa}{}^\kappa$  where  $D_\mu^C v_\nu \equiv \partial_\mu v_\nu - \Gamma_{\mu\nu}{}^\lambda v_\lambda$ ,  $D_\mu^C v^\lambda \equiv \partial_\mu v^\lambda + \Gamma_{\mu\nu}{}^\lambda v^\nu$ . The defect conservation laws (3) read

$$D_\mu^* G_\lambda{}^\mu + 2S^{\nu\lambda\kappa} G_{\kappa\nu}^C - \frac{1}{2} S^{\nu\kappa,\mu} R_{\lambda\mu\nu\kappa}^C = 0, \quad (18)$$

$$D^{*\mu} S_{\lambda\kappa,\mu} = G_{\lambda\kappa}^C - G_{\kappa\lambda}^C. \quad (19)$$

They are Bianchi identities ensuring the single-valuedness of observables, connection  $\Gamma_{\mu\nu}{}^\lambda$  and metric  $g_{\mu\nu}$ , via the integrability conditions  $[\partial_\sigma, \partial_\tau] \Gamma_{\mu\nu}{}^\lambda = 0$  and  $[\partial_\sigma, \partial_\tau] g_{\mu\nu} = 0$ .

In a four-dimensional Riemann-Cartan spacetime, the geometry is described by the direct generalizations of *translational* and *rotational* defect gauge fields  $h_{ij}$  and  $A_{ijk}$ , which are here the vierbein field  $h^\alpha{}_\mu$ , and the spin connection  $A_{\mu\alpha}{}^\beta$ . The square of the former is the metric  $g_{\mu\nu} = h^\alpha{}_\mu h_{\alpha\nu}$ . The latter is defined by the covariant derivative  $D_\lambda^C h_\beta{}^\mu = \partial_\lambda h_\beta{}^\mu - A_{\lambda\beta}{}^\gamma h_\gamma{}^\mu + \Gamma_{\lambda\nu}{}^\mu h_\beta{}^\nu \equiv D_\lambda^L h_\beta{}^\mu + \Gamma_{\lambda\nu}^C{}^\mu h_\beta{}^\nu$ . The field strength of  $A_{\mu\alpha}{}^\beta \equiv (A_\mu)_\alpha{}^\beta$

$$F_{\mu\nu\beta}{}^\gamma \equiv \{\partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu]\}_\lambda{}^\kappa, \quad (20)$$

determines the Cartan curvature  $R_{\mu\nu\lambda}^C{}^\kappa \equiv h^\beta{}_\lambda h_\gamma{}^\kappa F_{\mu\nu\beta}{}^\gamma$ . The field strength of  $h^\gamma{}_\nu$  is the torsion:

$$S_{\alpha\beta}{}^\gamma \equiv \frac{1}{2} h_\alpha{}^\mu h_\beta{}^\nu [D_\mu^L h_\nu{}^\gamma - (\mu \leftrightarrow \nu)]. \quad (21)$$

The relations (16), (18), and (19) follow from this.

8. The theory is gauge invariant under local Lorentz transformations as a direct consequence of the fact that the metric can alternatively be written as

$$g_{\mu\nu} = h^\gamma_\mu \Lambda^a_\gamma \Lambda_a^\beta h_{\beta\nu}, \quad (22)$$

where  $\Lambda_a^\beta$  is an *arbitrary local* Lorentz transformation, and that the Einstein-Hilbert Lagrangian  $\mathcal{L}_{\text{EH}} = -(1/2\kappa)R$  is *independent* of  $\Lambda^a_\alpha$ . The extra  $\Lambda_a^\beta$  transforms the gauge field  $A_{\mu\alpha}^\beta$  as

$$A_{\mu\alpha}^\beta \rightarrow A_{\mu\alpha}^\beta + \Delta A_{\mu\alpha}^\beta, \quad \Delta A_{\mu\alpha}^\beta \equiv \Lambda_a^\beta \partial_\mu \Lambda^a_\alpha. \quad (23)$$

At this point we are ready to introduce the *new* gauge invariance announced in the title: we allow  $\Lambda_a^\beta$  in Eq. (22) to be a *multivalued Lorentz transformation*. This is *not* integrable, so that  $\Delta A_{\mu\alpha}^\beta$  is a *nontrivial* gauge field. Indeed, the rotational field strength  $F_{\mu\nu\alpha}^\gamma$  can be expressed as  $F_{\mu\nu\alpha}^\gamma \equiv \Lambda_a^\gamma [\partial_\mu, \partial_\nu] \Lambda^a_\alpha \neq 0$  and yields a nonzero Cartan curvature  $R_{\mu\nu\lambda}^C \neq 0$ . The important observation is that a *multivalued*  $\Lambda^a_\alpha$  is able to *change the geometry* [19]. The right-hand side of (16) is *independent* of the vector field  $A_{\mu\alpha}^\beta$ . This allows us to shuffle torsion into Cartan curvature and back, fully or partially, by complete analogy with the defect transformations in two-dimensional crystals in Fig. 6. We may choose for  $A_{\mu\alpha}^\beta$  any function we like. For example we may choose it to make the torsion vanish, and  $A_{\mu\alpha}^\beta$  reduces to the usual spin connection of Einstein's gravity, the well-known combination of the objects of anholonomy

$$\Omega_{\mu\nu}^\lambda = \frac{1}{2} [h_\alpha^\lambda \partial_\mu h^\alpha_\nu - (\mu \leftrightarrow \nu)]. \quad (24)$$

In the opposite extreme  $A_{\mu\alpha}^\beta = 0$ , the Cartan curvature is zero, spacetime is teleparallel, and the Lagrangian is equal to the combination of torsion tensors:

$$\mathcal{L}_S = -\frac{1}{2\kappa} (-4D_\mu S^\mu + S_{\mu\nu\lambda} S^{\mu\nu\lambda} + 2S_{\mu\nu\lambda} S^{\mu\lambda\nu} - 4S^\mu S_\mu), \quad (25)$$

where  $S_\mu \equiv S_{\mu\nu}^\nu$ .

In any of the new gauges, the correct gravitational field equations are derived by extremizing the Einstein-Hilbert action

$$\mathcal{A}_{\text{EH}} = -\frac{1}{2\kappa} \int dx \sqrt{g} R^C + \int dx \sqrt{g} \mathcal{L}_S + \mathcal{A}_{\text{GF}}, \quad (26)$$

where  $\mathcal{A}_{\text{GF}}$  is a functional of  $h^\alpha_\mu$  and  $A_{\mu\alpha}^\beta$  fixing some convenient gauge. For  $\mathcal{A}_{\text{GF}} = \delta[A_{\mu\alpha}^\beta]$  this leads to the teleparallel theory, and for  $\mathcal{A}_{\text{GF}} = \delta[S_{\alpha,\beta,\gamma}[h^\alpha_\mu, A_{\mu\alpha}^\beta]]$  we re-obtain Einstein's original theory.

9. Adding matter fields of masses  $m$  to the Einstein Lagrangian, and varying with respect to  $h^\alpha_\mu$ , we find in the zero-torsion gauge the Einstein equation

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad (27)$$



where  $T_{\mu\nu}$  is the sum over the symmetric energy-momentum tensors of all matter fields. Each contains the canonical energy-momentum tensor  ${}^m\Theta_{\mu\nu}$  and the spin current densities  ${}^m\Sigma_{\mu\nu}{}^\lambda$  in the combination due to Belinfante [20],

$${}^mT_{\kappa\nu} = {}^m\Theta_{\kappa\nu} - \frac{1}{2}D^{*\mu} \left( {}^m\Sigma_{\kappa\nu,\mu} - {}^m\Sigma_{\nu\mu,\kappa} + {}^m\Sigma_{\mu\kappa,\nu} \right), \quad (28)$$

which is the matter analog of the defect relation (16).

The new gauge invariance of (16) has the physical consequence that the external gravitational field in the far-zone of a celestial body does not care whether angular momentum comes from rotation of matter or from internal spins. The off-diagonal elements of the metric in the far-zone, and thus the Lense-Thirring effect measured in [15], depend only on the total angular momentum  $J^\lambda = \int d^3x (x^\lambda T^{\mu 0} - x^\mu T^{\lambda 0})$ , which by the Belinfante relation (28) is the sum of orbital angular momentum  $L^\lambda = \int d^3x (x^\lambda \Theta^{\mu 0} - x^\mu \Theta^{\lambda 0})$  and spin  $S^\lambda = \int d^3x \Sigma^{\lambda\mu,0}$ . A star consisting of polarized matter has the same external gravitational field in the far-zone as a star rotating with the corresponding orbital angular momentum. This is the *universality of orbital momentum and intrinsic angular momentum* in gravitational physics observed in Ref. [10].

Since torsion is merely a *new-gauge degree* of freedom in describing a gravitational field, it cannot be detected experimentally, not even by spinning particles. A field with arbitrary spin may be coupled to gravity via the covariant derivative  $D_\mu \equiv \partial_\mu \mathbf{1} + \frac{i}{2} A_{\mu\alpha}{}^\beta \Sigma^\alpha{}_\beta$ , where  $\Sigma^\alpha{}_\beta$  are the generators of the Lorentz group, in the Dirac case  $\Sigma_{\alpha\beta} = \frac{i}{4}[\gamma^\alpha, \gamma_\beta]$ . But since the torsion is a tensor, we may equally well use an infinity of alternative covariant derivatives  $D_\mu^q \equiv \partial_\mu \mathbf{1} + \frac{i}{2} A_{\mu\alpha}^q{}^\beta \Sigma^\alpha{}_\beta$ , where  $A_{\mu\alpha}^q{}^\beta \equiv A_{\mu\alpha}{}^\beta - q K_{\mu\alpha}{}^\beta$ , and  $K_{\mu\alpha\beta} = h_\alpha{}^\nu h_\beta{}^\lambda K_{\mu\nu\lambda} \equiv h_\alpha{}^\nu h_\beta{}^\lambda (S_{\mu\nu\lambda} - S_{\nu\lambda\mu} + S_{\lambda\mu\nu})$ . Any coupling constant  $q$  is permitted by covariance. In order to see which  $q$  is physically correct we come back to the above-discussed photon mass problem, and consider the covariant electromagnetic field tensor  $F_{\mu\nu}^q \equiv D_\mu^q A_\nu - D_\nu^q A_\mu$ . Working out the covariant derivative we find  $\partial_\mu A_\nu - \partial_\nu A_\mu - 2(1-q)S_{\mu\nu}{}^\lambda A_\lambda$ , which shows that Maxwell Lagrangian  $-\frac{1}{4}F_{\mu\nu}^q F^{q\mu\nu}$  acquires a mass term, unless we fix the coupling strength to the value  $q = 1$ .

For this value of  $q$ , a little algebra [8, 9] shows that the torsion drops out from the gauge field  $A_{\mu\alpha}^q{}^\beta$ . This reduces to the good-old Fock-Ivanenko spin connection that has been used in Einstein gravity without torsion:

$$A_{\mu\alpha}^1{}^\beta = \bar{A}_{\mu\alpha}{}^\beta = h_\alpha{}^\nu h^{\beta\lambda} (\Omega_{\mu\nu\lambda} - \Omega_{\nu\lambda\mu} + \Omega_{\lambda\mu\nu}). \quad (29)$$

Having ensured that the photon does not couple to torsion, we must also prevent all other spinning baryonic matter to do so, to avoid giving a mass to the photons via virtual processes of the type discussed above and illustrated in Fig. 1.

**10.** How about the motion of a spinless point particle in the infinitely many different descriptions of the same theory? Since the metric  $g_{\mu\nu} = h^\gamma{}_\mu \Lambda^a{}_\gamma \Lambda_a{}^\beta h_{\beta\nu}$  is independent of the local Lorentz transformations  $\Lambda^a{}_\alpha$ , and the action  $\mathcal{A} = -mc \int ds = -mc \int (g_{\mu\nu} dx^\mu dx^\nu)^{1/2}$  depends only on  $g_{\mu\nu}$ , the trajectories are geodesics for all  $\Lambda_a{}^\beta$ . The same result can of course be obtained by integrating the local conservation law of the total energy-momentum tensor  $T_{\mu\nu}$  along a thin world-tube.

A spinning particle “sees” the gauge field of Lorentz transformations  $A_{\mu\alpha}^{\beta}$ , but it does so only via the  $q = 1$ -version (29). This contains only the vierbein fields, not the torsion, and is invariant under the multivalued version of the gauge transformation (23). Hence the motion of a spinning particle is blind the torsion field, which can therefore not be detected by any experiment.

**11.** What we have done can be understood better by a simple analogy. Instead of Einstein’s theory, we consider a model of a real field  $\rho$  with an Euclidean Lagrangian  $\mathcal{L} = (\partial_\mu \rho)^2 - \rho^2 + \rho^4$  and a partition function  $Z = \int \mathcal{D}\rho e^{-\int dx \mathcal{L}}$ . The field  $\rho$  is the analog of the metric  $g_{\mu\nu}$ . We may now trivially introduce an extra gauge structure by re-expressing the Lagrangian in terms of a complex field  $\psi = e^{i\theta} \rho$  and a gauge field  $A_\mu$  as  $\tilde{\mathcal{L}} = |(\partial_\mu - iA_\mu)\psi|^2 - |\psi|^2 + |\psi|^4$ . Now we form the partition function  $\bar{Z} = \int \mathcal{D}\psi \mathcal{D}\psi^* \mathcal{D}A_\mu \Phi e^{-\int dx \tilde{\mathcal{L}}}$ , where  $\Phi$  is an arbitrary gauge-fixing functional multiplied by the associated Faddeev-Popov determinant. The classical physics described by the new  $\bar{Z}$  is exactly the same as that of the original  $Z$ . Obviously there is no way of observing  $A_\mu$ . The partition function  $Z$  plays the role of Einstein’s theory, whereas  $\bar{Z}$  represents its reformulation in terms of a gauge field. The decomposition  $\rho = \psi^* \psi = (\rho e^{-i\theta})(e^{i\theta} \rho)$  is the analog of the decomposition (22).

**12.** Higher gradient terms in elastic energy of the world crystal will generate an extra action  $\mathcal{A}_{A_\mu}$  of the gauge field  $A_{\mu\alpha}^{\beta}$  [21]. This would, in general, violate the new symmetry discussed above and give torsion a life of its own. However, as long as the gravitational effects of spinning constituents in celestial bodies are suppressed with respect to that of the orbital angular momenta by many orders of magnitude, there is not much sense in conjecturing explicit forms of  $\mathcal{A}_{A_\mu}$ , unless we want to compete with string theory in setting up an ultimate *theory of everything* as a substitute of religion.

**13.** In summary, we have shown that if the Einstein-Hilbert Lagrangian is expressed in terms of the translational and rotational gauge fields  $h^\alpha_\mu$  and  $A_{\mu\alpha}^{\beta}$ , the Cartan curvature can be converted to torsion and back, totally or partially, by a *new type of multivalued gauge transformation* in Riemann-Cartan spacetime, a *hypergauge transformation*. In this general formulation, Einstein’s original theory is obtained by going to the zero-torsion hypergauge, while his teleparallel theory is in the hypergauge in which the Cartan curvature tensor vanishes. But any intermediate choice of the field  $A_{\mu\alpha}^{\beta}$  is also allowed.

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