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QUARKS AND REGGEONS*

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Abstract: We propose a new algebra which will hopefully contribute to the joint understanding of electromagnetic and weak structure functions and of Regge couplings. It involves an infinite number of form factors of arbitrary complex spin J (with $\mathrm{Re}(J-1) > 0$) and definite signature. The algebra holds exactly in a quark-gluon model when quantized canonically on equal light fronts. If all form factors have $k = 0$ and $J = 1$, the algebra closes and becomes U(6) × U(6), with the connecting subgroups being associated with pure quarks and pure antiquark currents, respectively. This doubling of the group leads to exact exchange degeneracy if baryons and mesons consist of qqq and $q\bar{q}$ wave functions only.

To some lowest approximation, Regge couplings can be calculated by representing U(6) × U(6) on properly mixed quark wave functions. With the mixing parameters fixed by the chiral subgroup, these couplings are determined up to an overall normalization.

As an illustration, the couplings of the trajectories ρ, ω; A_2, f; A_1, D; Z, Z_{sg}; π, η;B, H trajectories are estimated for the meson resonsnces ρ, ω; π, η; A_2, f; A_1,D; B, H; A_0, σ. The trajectories π, η; B, H are not in the algebra initially but can be inferred from an extension of PCAC to the bilocal currents ("PCBC").

1. Introduction

With increasing accelerator energies, Regge exchanges have begun to play a dominant role in strong interaction physics. This has led to a number of theoretical investigations both on the more detailed structure and the exchanged singularities [1] as well as on possible couplings of these singularities among each other [2].

A large amount of experimental information on Regge couplings will come from the peripheral production of higher resonances. Even though many of these data have been analyzed from the phenomenological point of view [3], there has been little progress in their understanding in more fundamental terms.

It is the purpose of this paper to develop an algebraic approach to Regge couplings that makes optimal use of what we know about the quark structure of

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hadrons from deep-inelastic lepton hadron scattering. As it will turn out, universal mixing schemes of chiral SU(3) × SU(3) charge algebra on quark wave functions classified according to U(6) × O(3) provide essentially all the information needed for a lowest order understanding of these couplings. Mixing schemes of this kind have been proposed and studied in the literature some time ago [3, 4] and have recently been reinvestigated and extended in great detail [5].

The idea that Regge couplings might be amenable to some kind of algebraic treatment has been around [6] ever since the discovery of current algebra. However, it was not until recently with progress in multi-reggeon and in quark-gluon theories, that the nature of this algebra could be well defined [7] and that connections with more familiar current structures were recognized [8]. It was finally becoming clear that currents and Regge couplings could be combined in one large algebraic scheme [9] and thus turn into an object of representation theory in terms of quarks.

Our starting point will be the algebra of bilocal form factors of all "good" U(6) charges of the standard quark model quantized canonically on light fronts. The philosophy and basic properties of their model as well as its experimental implications on deep-inelastic electron and neutrino structure functions are all assumed to be well-known [10]. There are some arguments based on perturbation theory in the deep-euclidean limit suggesting the algebra to be hampered by logarithmic factors [11]. However, such factors do not appear to be present in experiment*. Thus it is possible that they are either a false property of the model or there are flaws in the arguments *via* the deep-euclidean limit. We shall take the standpoint that our basic algebra is, in fact, true exactly.

The algebra of bilocal form factors can be transformed in a straightforward manner into an algebra of signatured form factors [9]. It is this algebra that offers direct access to Regge couplings.

The introduction of signature leads to a doubling of the original U(6) group and is the group theoretic basis of exchange degeneracy of ρ, A_2; ω, f, etc., trajectories.

As an application, a large meson scheme will be presented which gives rise to many predictions on the peripheral production of meson resonances while incorporating all known results of current algebra.

2. The algebra of signatured from factors

According to the philosophy presented in the introduction it will be assumed that good components of quark fields commute canonically on equal light fronts [10]. As a consequence, the algebra U(6) of light-like vector, axial vector and tensor charges can naturally be extended to bilocal operators [10]. For the vector current this bilocal extension reads:

* Unless the strong coupling constant is assumed to be so small that it is hard to conceive of strong binding forces among quarks [12].

$$\hat{F}_a(k,\zeta) \equiv \int \mathrm{d}^2R^{\perp}\mathrm{d}R^{-}\,\mathrm{e}^{ikR}\,\bar{\psi}(R+\tfrac{1}{2}\zeta)\,\gamma^{+}\,\tfrac{1}{2}\lambda_a\,\psi(R-\tfrac{1}{2}\zeta)\,. \tag{1}$$

For the remaining operators of U(6), $\hat{F}_{5a}$, $\hat{F}_{1a}$, and $\hat{F}_{2a}$ one has to replace γ^+ by $\gamma^+\gamma_5$, $-i\gamma^+\gamma^2$, $i\gamma^+\gamma^1$, respectively. It is understood that ζ has only a ζ^- component*. The matrix elements of these operators,

$$2p^+\,(2\pi)^3\delta^{(2)}\,(p^{\perp\prime}-p^{\perp}-k)\,\delta\,(p^{+\prime}-p^+)F_a\,(k,p^+\zeta^-) = \langle p'|\hat{F}_a(k,\zeta)\,|p\rangle \tag{2}$$

are called bilocal form factors and satisfy the algebra [10]

$$[F_a(k,z),F_b(k',z')] = i\,f_{abc}\,F_c(k+k',z+z')\,. \tag{3}$$

Corresponding commutators hold for the remaining bilocal form factors F_{5a}, F_{1a}, F_{2a}.

Expansion of $F_a(k,z)$ in a power series

$$F_a(k,z) = \sum_{J=1}^{\infty} \frac{(iz)^{J-1}}{(J-1)!}\,F_a^J(k)\,, \tag{4}$$

leads to form factors of definite spin J. In the case of vector and axial form factors $F^J(k)$, $F_5^J(k)$, these can be measured in deep-inelastic processes

$$\text{“}\gamma\text{”}(q+k) + \beta(p') \leftarrow \text{“}\gamma\text{”}(q) + \alpha(p)\,, \tag{5}$$

with $q^2 \to \infty$ at fixed $\xi = -q^2/(p'+p)q$, $k^+ = 0$, $k^{\perp} = (k^1, k^2)$. Here “γ” stands for a photon or W meson emitted by electrons or neutrinos, respectively. If $F_a(k,\xi)$ denotes the structure functions of such processes, then $F_a^J(k)$ is simply given by the moments

$$F_a^J(k) \equiv \int_{-1}^{1} \xi^{J-1}\,F(k,\xi)\mathrm{d}\xi\,. \tag{6}$$

It has been pointed out recently, that after properly introducing signature the form factors (6) can be continued analytically in spin J. For this one splits the structure functions in symmetric and antisymmetric parts

$$F(k,\xi) = \tfrac{1}{2}\,(F^{\mathrm{S}}\,(k,\xi) + F^{\mathrm{A}}\,(k,\xi))\,,$$

and divides $F^{\mathrm{S}}_{\mathrm{A}}(k,\xi)$ in parts which are non-zero only for $\xi \in [0,1]$:

$$F^{\mathrm{S}}_{\mathrm{A}}(k,\xi) = F^{\mp}\,(k,\xi) \pm F^{\mp}\,(k,-\xi)\,. \tag{7}$$

* We use $x^+ = \frac{1}{2}(x^0 + x^3)$, $x^- = x^0 - x^3$, $x^{\perp} = (x^1, x^2)$.

The signatured form factors are now defined as:

$$F^{J\mp}(k) = \int_{-1}^{1} \xi^{J-1}\, F^{\mp}(k, \xi)\mathrm{d}\xi = \int_{0}^{1} \xi^{J-1}\, F^{\overset{\mathrm{S}}{\mathrm{A}}}(k, \xi)\mathrm{d}\xi\,, \tag{8}$$

and exist as they stand for any complex J with $\mathrm{Re}(J-1) > 0$. For large J in this region $F^{J\mp}$ are expected [9] to fall off at least as $|J|^{-4}$.

Our second basic assumption is now that Regge poles survive the scaling limit. If this is so, there will be poles in the signatured form factor

$$F^{J\mp}(k) = \sum_i \frac{R^i(k)}{J-\alpha_i(k)} + \text{cuts}\,, \tag{9}$$

where $R^i(k)$ are the corresponding Regge couplings. Since the invariant momentum transfer is $-k^2 < 0$, all trajectories $\alpha_i(k)$ will be below the unitarity limit $\alpha(0) \leqslant 1$.

Notice that in the complex $\omega \equiv 1-J$ plane the singularities are very similar to the spectrum of a non-relativistic Hamiltonian. Thus $F^{J\mp}(k)$ is apparently closely related to the phenomenological fields $\phi(\omega, k)$ used in the Gribov calculus for reggeons [1].

At this point one should also point out that, certainly, the Regge couplings $R^i(k)$ are not quite the purely hadronic ones which we would like to know. They contain, in addition, the couplings to the two deep-inelastic photons at the upper vertex in the process (5). If we assume, however, factorization at each Regge pole, then hadronic couplings *do* become acessible up to an unknown overall normalization.

The leading trajectories in $F^{J\mp}(k)$, $F^{J\mp}_{1,2}(k)$, and $F^{J\mp}_5(k)$ will be ρ, ω; A_2, f and A_1, D; Z, Z_{sg} together with their SU(3) partners, respectively. In addition $F^{J+}_{0,8}(k)$ and $F^{J+}_{1,2;0,8}(k)$ may carry a singularity due to diffractive effects (pomeron exchange!). In the case of the charges ($k = 0$), $F^{J\mp}(0)$ describes the helicity conserving*, $F^{J\mp}_{1,2}(0)$ helicity flip couplings of ρ, ω; A_2, f trajectories and possibly of the pomeron.

The important point is now that the algebra (3) implies an algebra for these signatured form factors:

$$[F^{J\eta}_a(k),\ F^{J',\eta'}_b(k')] = if_{abc}\, F^{J+J'-1,-\eta\eta'}_c(k+k')\,, \tag{10}$$

with corresponding commutators for the other form factors. The special case $J, \eta = J', \eta' = 1,-$ coincides with the old U(6) algebra, with $F^{1-}_a(k)$, $F^{1-}_{5,a}(k)$ forming the well known chiral SU(3) $\times$ SU(3) subalgebra.

The introduction of signature has the important consequence that there exists a

* When we talk of helicity we mean the infinite momentum helicity in z direction. This is equivalent to s-channel helicity when scattering takes place in forward direction.

new extension of U(6). It consists of operators of $J = J' = 1$ with both signatures. This doubling will turn out to be the group theoretic basis of the experimentally observed exchange degeneracy of reggeons.

It is obvious that the whole algebra (10) can now be continued analytically to arbitrary J, J' as long as the commutators exist. Some interesting sum rules emerge which have been discussed previously [9].

For example, if J' runs into a Regge pole $R^{i,\eta}(k)$ at $\alpha^i(k)$, $F^{J,\mp}(0)$ turns out to be a daughter lowering operator by $J-1$ units:

$$[F_a^{J,\mp}(0), R_b^{i,\eta}(k)] = if_{abc}\, R_c^{i+J-1,\mp\eta}(k)\,.$$

If there is no such daughter trajectory which can be singular on the right-hand side a superconvergence relation emerges:

$$[F_a^{J,\mp}(0), R^{i,\eta}(k)] = 0\,. \tag{11}$$

Similarly, taking both J and J' to a Regge pole $\alpha_i(k)$ and $\alpha_j(k')$ leads to a superconvergence relation for reggeons

$$[R^{i\eta}(k), R^{j,\eta'}(k')] = 0\,. \tag{12}$$

Triple Regge considerations permit an estimate for the range of momentum transfers k, k' in which (11) and (12) are expected to be valid. One finds that

$$\alpha_k(k+k') - \alpha_i(k) - J + 1 < 0\,, \tag{13}$$

for (11) and

$$\Delta_k^{ij} \equiv \alpha_k(k+k') - \alpha_i(k) - \alpha_j(k') + 1 < 0\,,$$

for (12) where $\alpha_k(k+k')$ is the leading trajectory in the corresponding scattering process. If these conditions are not fulfilled the superconvergence relations will turn into algebraic versions of finite energy sum rules. For example, (12) becomes

$$[R_a^i(k), R^j(k')] = if_{abc} \sum_k g_k^{ij} \frac{N^{\Delta^{ij}}}{\Delta_k^{ij}}\, R_c^k(k+k')$$

where g_k^{ij} are triple Regge couplings.

As far as daughter trajectories can be neglected in Σ_k, the triple Regge couplings satisfy the approximate U(6) × U(6) relations*

$$g_\rho^{\rho\rho} \approx g_{A_2}^{\rho A_2} \approx g_\pi^{\rho\pi} \approx g_B^{\rho B} \text{ etc.}$$

* Due to the consistency conditions following from the Jacobi identity.

For kinematical reasons, such neglect cannot be true† for a larger range of k, k'. It may, however, hold at $k = k' = 0$ when saturation should be fast.

In this work we shall be dealing with the algebra (10) only for the charges $k = k' = 0$. In this case, the U(6) × U(6) subgroup with $J = J' = 1$ seems to run into a difficulty††: the operators $F^{1+}_{0,8}(0)$, $F^{1+}_{1,2;0,8}(0)$ do apparently not exist due to the pomeron singularity at $J = 1$.

The superconvergence relation (11), however, ensures us immediately that this difficulty does not really arise. In fact, the pomeron singularity turns out to be a complete singlet under U(6) × U(6). For example, using the commutator

$$[F^{1\mp}_4(0), F^{J+}_8(0)] = -\sqrt{\tfrac{3}{2}}\, i\, F^{J,\pm}_5(0)\,,$$

and letting $J \to 1$ we see that the pomeron residue commutes with $F^{1\mp}_4(0)$. In order to make sure that this is valid we confirm that condition (13) is respected: $0.25 - 1 - 1 + 1 < 0$.

A similar argument holds for the commutators with all other generators*. Thus only $F^{J+}_0(0)$ can have the pomeron singularity at $J = 1$**.

If we remove the pomeron singularity in $F^{1+}_0(0)$ we can deal with the full algebra of signatured form factor (10) at $k = k' = 0$ and assume only true Regge singularities to appear in an expansion (9).

Let us now turn to the practical problem of calculating Regge couplings. For this we shall work completely within a narrow resonance approximation. At this level, also the trajectories are purely real and involve no cuts. There is no pomeron coupling in $F^{1+}_0(0)$ from the beginning. The program we have in mind is the following. As a first step we shall represent only the algebra U(6) × U(6) formed by the charges $F^{1+}_a(0)$ etc. Then Regge couplings can be estimated by using the closest pole approximation in the expansion (9):

$$F^{1+}(0) \sim \frac{R^{\rho,A_2}}{1-\alpha_{\rho,A_2}(0)}\,, \quad F^{1\mp}_{\frac{1}{2}}(0) \sim \frac{R_{\frac{1}{2}}^{\rho,A_2}}{1-\alpha_{\rho,A_2}(0)}\,. \tag{14}$$

Since $\alpha_{\rho,A_2}(0) \sim \frac{1}{2}$ and the next lower trajectory follows at $-\frac{1}{2}$, this approximation should be good to at least 30%.

The axial vector trajectories A_1, D and their exchange degenerate partners cannot as reliably be estimated from (14). Their intercept is $\alpha_{A_1,Z}(0) \approx 0$ and the next lower term will bring corrections up to 50%. It is gratifying to note that these trajectories have played no significant role in any phenomenological analysis so we shall discard them altogether.

† I thank M. Suzuki for a discussion of this point.

†† I thank Murray Gell-Mann for pointing this difficulty out and helping me to resolve it.

* Notice that the purely hadronic couplings of the pomeron could still be non-signlet. Since their decoupling in $F^{J+}(0)$ may be due to the two photon vertex in the process (5).

** We assume, the pomeron to be a strong singularity in the J plane with $F^{J+} \to \infty$ as $J \to 1$.

Notice, on the other hand, that the axial charge and its exchange degenerate partners do carry an important information on the couplings of π, η and B, H trajectories respectively. It is well known that on the basis of PCAC, the matrix elements of $F_5^{1-}(0)$ are related to pionic couplings at $q^2 = 0$ *via*

$$\frac{i}{f_\pi}(m_\beta{}^2 - m_\alpha{}^2)\,(F_5^{1-}(0))_{\beta\alpha} = \langle\beta|j_\pi(0)|\alpha\rangle|_{q^2=0}\,. \tag{15}$$

But the intercept of the poin trajectory is almost zero such that the poin does not reggeize much. Thus the same relation also holds for the coupling of the poin trajectory. By exchange degeneracy, $F_5^{1+}(0)$ should give the couplings of the B trajectory. Alternatively one may argue that the divergence for the bilocal currents [10] is dominated by the leading trajectories ("PCBC"). This gives a PCAC type of relation for the charges of both signatures with π, η and B, H trajectories appearing on the right-hand side.

3. Quark representation of U(6) × U(6)

Let us split the quark field $\psi(x)$ in good and bad components according to*

$$\psi(x) = \tfrac{1}{2}(1+\gamma^0\gamma^3)\psi^{(+)}(x) + \tfrac{1}{2}(1-\gamma^0\gamma^3)\psi^{(-)}(x)$$

$$= \begin{pmatrix} \psi_1^{(+)} \\ \psi_1^{(-)} \\ \psi_2^{(-)} \\ \psi_2^{(+)} \end{pmatrix},$$

and expand the good components in terms of quark and antiquark creation and annihilation operators of infinite momentum helicity s:

$$\psi^{(+)}(x) = \sum_s \int \frac{d^2p^\perp dp^+}{(2\pi)^{\frac{3}{2}}} \Big\{ e^{-i(p^+x^- - p^\perp x^\perp)}\,\omega(s)Q(p^\perp, p^+, s)$$
$$+ e^{i(p^+x^- - p^\perp x^\perp)}\,\omega(-s)\bar{Q}^+(p^\perp, p^+, s)\Big\}. \tag{16}$$

* We use

$$\gamma^\mu = \begin{pmatrix} 0 & \tilde{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix}$$

as Dirac matrices with $\tilde{\sigma}^\mu = (\sigma^0, -\sigma^i)$.

Here

$$\omega(\tfrac{1}{2}) = \begin{pmatrix}1\\0\end{pmatrix} \text{ and } \omega(-\tfrac{1}{2}) = \begin{pmatrix}0\\1\end{pmatrix}$$

such that charge conjugation is represented by

$$\hat{C}\,\omega^{(+)}(x)\,\hat{C}^{-1} = \sigma_1\,\psi^{(+)}(x)\,. \tag{17}$$

The trivial SU(3) indices have been suppressed. If we introduce the shorter notation

$$Q(p^\perp, p^+) \equiv \sum_{s=-\frac{1}{2}}^{\frac{1}{2}} \omega(s) Q(p^\perp, p^+, s)\,,\ \bar{Q}(p^\perp, p^+) \equiv \sum_{s=-\frac{1}{2}}^{\frac{1}{2}} \omega(-s)\bar{Q}(p^\perp, p^+, s)\,,$$

then the operators corresponding to the form factors $F^J(k)$, $F_5^J(k)$ can be written explicitly as

$$\hat{F}_a^J(k) = \frac{1}{(P^+)^J}\int d^2p^\perp\, dp^+ \Big\{ (p^+)^{J-1} Q^+(p^\perp + k, p^+)\,\tfrac{1}{2}\lambda_a\, Q(p^\perp, p^+)$$

$$+ (-p^+)^{J-1}\,\bar{Q}^+(p^\perp + k, p^+)\,(-\tfrac{1}{2}\lambda_a{}^*)\,\bar{Q}(p^\perp, p^+)\Big\}\,,$$

$$\hat{F}_{5a}^J(k) = \frac{1}{(P^+)^J}\int d^2p^\perp\, dp^+ \Big\{ (p^+)^{J-1} Q^+(p^\perp + k, p^+)\,\tfrac{1}{2}\lambda_a\,\sigma_3\, Q(p^\perp, p^+)$$

$$+ (-p^+)^{J-1}\,\bar{Q}^+(p^\perp + k, p^+)\,\tfrac{1}{2}\lambda_a{}^*\,\sigma_3\,\bar{Q}(p^\perp, p^+)\Big\}\,. \tag{18}$$

Signatured operators $\hat{F}^{J\mp}(k)$, $\hat{F}_5^{J\mp}(k)$ are obtained by changing $(-p^+)^{J-1} \to \pm (p^+)^{J-1}$. Due to the spectral condition, the integral can run only over the interval $p^+ \epsilon (0, P^+)$ and this is what permits direct analytic continuation [9] of the operator* in spin J. Similar expressions hold for $F_5^{J\mp}(k)$ and $F_1^{J\mp}(k)$.

In this work we shall investigate the subalgebra U(6) × U(6) of the charges with $k = 0$. If we define

$$\mathrm{O}^J(a \times 1 + 1 \times b) \equiv \frac{1}{(P^+)^J}\int d^2p^\perp\, dp^+ (p^+)^{J-1} \Big\{ Q^+(p^\perp, p^+)\, a Q(p^\perp, p^+)$$

$$+ \bar{Q}^+(p^\perp, p^+) b \bar{Q}(p^\perp, p^+)\Big\}\,, \tag{19}$$

* The analytic continuation to Regge operators has been studied from a dynamical point of view in a recent preprint by Brandt [14].

they can be written as

$$\hat{F}^{1\mp}(0) = O^1\,(\tfrac{1}{2}\lambda \times 1 \mp 1 \times \tfrac{1}{2}\lambda^*);\; C = \mp, S = \mp\,,$$

$$\hat{F}_5^{1\mp}(0) = O^1\,(\tfrac{1}{2}\lambda\,\sigma^3 \times 1 \pm 1 \times \tfrac{1}{2}\lambda^*\,\sigma^3);\; C = \pm, S = \mp\,,$$

$$\hat{F}_{\frac{1}{2}}^{1\mp}(0) = O^1\,(\tfrac{1}{2}\lambda\,\sigma^{\frac{1}{2}} \times 1 \mp 1 \times \tfrac{1}{2}\lambda^*\,\sigma^{\frac{1}{2}});\; C = \mp, S = \mp\,. \tag{20}$$

Because of their quantum numbers C, S the leading trajectories of these operations are

$$F_a^{1\mp}:\quad \rho^{(0)}, \omega^{(0)}; A_2^{(0)}, f^{(0)}\,,$$

$$F_{5_a}^{1\mp}:\quad A_1^{(0)}, D^{(0)}; Z^{(0)}, Z_{sg}^{(0)},$$

$$F_{1\pm i2,a}^{1\mp}:\ \rho^{(\pm)}, \omega^{(\pm)}; A_2^{(\pm)}, f^{(\pm)}\,,$$

with the corresponding U(3) partners. The superscript denotes the helicity properties ((0) = non-flip, (±) = flip up or down).

In order to find a representation of the charges (22) all we have to know are the quark wave functions of the resonances. These wave functions have been determined some time ago in saturation schemes of the chiral U(3) × U(3) sub-algebra consisting of $F_a^{1-}(0), F_{a,5}^{1-}(0)$ [3–5].

Such saturation schemes [3–5] start out with quark model wave functions of the type **35**, L = 0, 1, 2, . . . for mesons and **56**, L = 0; 70, L = 1; 56, L = 0, 2, for baryons. They directly provide a reducible representation of U(3) × U(3) formed by the matrices $\frac{1}{2}\lambda \times 1 - 1 \times \frac{1}{2}\lambda^*$ and $\frac{1}{2}\lambda\,\sigma_3 \times 1 + 1 \times \frac{1}{2}\lambda^*\,\sigma_3$. At this level there are no transitions between different orbital states. In order to obtain such transitions, a universal mixing operator has been proposed and all mixing parameters have been fixed [3–5]. Since PCAC relates $F_5^{1-}(0)$ to the pionic couplings according to (14), the outcome of such mixing schemes has been a large set of pionic decay widths of meson and baryon resonances. The experimentallly known widths are all in good agreement with the predictions. We refer the reader to the original works for a detailed description of these results.

The point is now that exactly the same wave functions allow for a direct extension of the chiral group to the whole signatured group U(6) × U(6).

In the saturation scheme baryons are represented by pure qqq, mesons by pure qq wave functions*. An immediate consequence is exact exchange degeneracy. Thus

* In fact there are small admixtures of q̄q, q̄qq̄q, etc., wave functions. They should only be seen by F_0^{J+} which contains the pomeron. This operator breaks exchange degeneracy.

one finds for baryons

$$F^{J+}_{\beta\alpha}(k) \equiv F^{J-}_{\beta\alpha}(k) \,, \tag{21}$$

which holds for all k. From equation (9) we see that this implies equality of all trajectories

$$\alpha^{i+}(k) = \alpha^{i-}(k) \,, \tag{22}$$

as well as all residues $R^{i+}_a(k) = R^{i-}_a(k)$. For mesons one has again (22) with certain cross relations of the residues of opposite signature. These will be presented in detail in sect. 4.

4. The meson scheme

For simplicity, we shall not take into account the full SU(3) symmetry but shall drop strangeness and restrict ourselves to the subgroup U(4) × U(4) of U(6) × U(6). Since purely mesonic couplings possess only trivial D/F ratios due to charge conjugation, we do not lose much information by proceeding this way. As announced before, we shall perform a saturation in terms of narrow resonances only. Then no diffractive effects should arise. The leading singularity in $F^{J+}_0(0)$ will directly be the f trajectory. It goes without saying that all SU(4) singlets are ideal mixtures of SU(3) octets and singlets (i.e., those with no strange quarks.)

With the isospin subgroup being self-adjoint, it is useful to perform a rotation $e^{i\pi I_2}$ on the antiparticles bringing $-\tau^*$ into τ. Then the operators (20) can be written, together with their leading trajectories, as

$$\begin{aligned}
&F^{1\mp}_a(0) = O^1\,(\tfrac{1}{2}\tau_a \times 1 \pm 1 \times \tfrac{1}{2}\tau_a);\; \rho^{(0)}, \mathrm{A}_2{}^{(0)} \,,\\
&F^{1\mp}_0(0) = O^1\,(\tfrac{1}{2} \times 1 \mp 1 \times \tfrac{1}{2});\; \omega^{(0)}, \mathrm{f}^{(0)} \,,\\
&F^{1\mp}_{5a}(0) = O^1\,(\tfrac{1}{2}\tau_a\,\sigma^3 \times 1 \mp 1 \times \tfrac{1}{2}\tau_a\,\sigma^3);\; \mathrm{A}_1{}^{(0)}, \mathrm{Z}^{(0)} \,,\\
&F^{1\mp}_{5_0}(0) = O^1\,(\tfrac{1}{2}\sigma^3 \times 1 \pm 1 \times \tfrac{1}{2}\sigma^3);\; \mathrm{D}^{(0)}, \mathrm{Z}^{(0)}_{\mathrm{sg}} ,\\
&F^{1\mp}_{\pm a}(0) = O^1\,(\tfrac{1}{2}\tau_a\,\sigma^\pm \times 1 \pm 1 \times \tfrac{1}{2}\tau_a\,\sigma^\pm);\; \rho^{(\pm)}, \mathrm{A}_2{}^{(\pm)} \,,\\
&F^{1\mp}_{\pm 0}(0) = O^1\,(\tfrac{1}{2}\sigma^\pm \times 1 \mp 1 \times \tfrac{1}{2}\sigma^\pm);\; \omega^{(\pm)}, \mathrm{f}^{(\pm)} \,.
\end{aligned} \tag{23}$$

The basic quark representation is (4, 1)

$$p\uparrow = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}, \quad p\downarrow = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix},$$

$$n\uparrow = \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}, \quad n\downarrow = \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}, \tag{24}$$

while antiquarks transform according to $(1, \bar{4})$:

$$(-\bar{n})^{\uparrow} = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}, \quad (-\bar{n})^{\downarrow} = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix},$$

$$\bar{p}^{\uparrow} = \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}, \quad \bar{p}^{\downarrow} = \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}. \tag{25}$$

Meson states are assigned to $(4, \bar{4})$ with different orbital wave functions. The ground state mesons can be listed according to the different helicities as*

$h = 1$

$$\rho_+^{(1)} = -\,p^{\uparrow}\bar{n}^{\uparrow}$$

$$\rho_0^{(1)} = \frac{1}{\sqrt{2}}(-n^{\uparrow}\bar{n}^{\uparrow} + p^{\uparrow}\bar{p}^{\uparrow})$$

$$\rho_-^{(1)} = n^{\uparrow}\bar{p}^{\uparrow}$$

$$\omega^{(1)} = \frac{1}{\sqrt{2}}(n^{\uparrow}\bar{n}^{\uparrow} + p^{\uparrow}\bar{p}^{\uparrow})$$

$h = 0$

$$\rho_+^{(0)} = \frac{1}{\sqrt{2}}(-p^{\downarrow}\bar{n}^{\uparrow} - p^{\uparrow}\bar{n}^{\downarrow})$$

$$\rho_0^{(0)} = \tfrac{1}{2}(-n^{\downarrow}\bar{n}^{\uparrow} - n^{\uparrow}\bar{n}^{\downarrow} + p^{\downarrow}\bar{p}^{\uparrow} + p^{\uparrow}\bar{p}^{\downarrow})$$

$$\rho_-^{(0)} = \frac{1}{\sqrt{2}}(n^{\downarrow}\bar{p}^{\uparrow} + n^{\uparrow}\bar{p}^{\downarrow})$$

$$\omega^{(0)} = \tfrac{1}{2}(n^{\downarrow}\bar{n}^{\uparrow} + n^{\uparrow}\bar{n}^{\downarrow} + p^{\downarrow}\bar{p}^{\uparrow} + p^{\uparrow}\bar{p}^{\downarrow})$$

$$\pi_+ = \frac{1}{\sqrt{2}}(p^{\downarrow}\bar{n}^{\uparrow} - p^{\uparrow}\bar{n}^{\downarrow})$$

$$\pi_0 = \tfrac{1}{2}(n^{\downarrow}\bar{n}^{\uparrow} - n^{\uparrow}\bar{n}^{\downarrow} - p^{\downarrow}\bar{p}^{\uparrow} + p^{\uparrow}\bar{p}^{\downarrow})$$

$$\pi_- = \frac{1}{\sqrt{2}}(-n^{\downarrow}\bar{p}^{\uparrow} + n^{\uparrow}\bar{p}^{\downarrow}) \tag{26}$$

$$\eta = \tfrac{1}{2}(-n^{\downarrow}\bar{n}^{\uparrow} + n^{\uparrow}\bar{n}^{\downarrow} - p^{\downarrow}\bar{p}^{\uparrow} + p^{\uparrow}\bar{p}^{\downarrow}).$$

Coupling these with a p wave orbital quantum $l^{(1)}, l^{(0)}, l^{(-1)}$ yields the first excited positive parity states:

* The subscripts denote charge states. The $h = -1$ states are obtained by applying $\frac{1}{2}\sigma^- \times 1 + 1 \times \frac{1}{2}\sigma^-$ to the $h = 0$ states.

$$
\begin{array}{lll}
h=2 & h=1 & h=0 \\
A_2=\rho^{(1)}l^{(1)} & A_2=\frac{1}{\sqrt{2}}(\rho^{(1)}l^{(0)}+\rho^{(0)}l^{(1)}) & A_2=\frac{1}{\sqrt{6}}(\rho^{(1)}l^{(-1)}+\rho^{(-1)}l^{(1)}+2\rho^{(0)}l^{(0)}) \\
\mathrm{f}\ =\omega^{(1)}l^{(1)} & \mathrm{f}\ =\frac{1}{\sqrt{2}}(\omega^{(1)}l^{(0)}+\omega^{(0)}l^{(1)}) & \mathrm{f}\ =\frac{1}{\sqrt{6}}(\omega^{(1)}l^{(-1)}+\omega^{(-1)}l^{(1)}+2\omega^{(0)}l^{(0)}) \\
 & A_1=\frac{1}{\sqrt{2}}(\rho^{(1)}l^{(0)}-\rho^{(0)}l^{(1)}) & A_1=\frac{1}{\sqrt{2}}(\rho^{(1)}l^{(-1)}-\rho^{(-1)}l^{(1)}) \\
 & \mathrm{D}\ =\frac{1}{\sqrt{2}}(\omega^{(1)}l^{(0)}-\omega^{(0)}l^{(1)}) & \mathrm{D}\ =\frac{1}{\sqrt{2}}(\omega^{(1)}l^{(-1)}-\omega^{(-1)}l^{(1)}) \\
 & \mathrm{B}\ =\pi\, l^{(1)} & \mathrm{B}\ =\pi\, l^{(0)} \\
 & \mathrm{H}\ =\eta\, l^{(1)} & \mathrm{H}\ =\eta\, l^{(0)} \\
 & & A_0=\frac{1}{\sqrt{3}}(\rho^{(1)}l^{(-1)}+\rho^{(-1)}l^{(1)}-\rho^{(0)}l^{(0)}) \\
 & & \sigma\ =\frac{1}{\sqrt{3}}(\omega^{(1)}l^{(-1)}+\omega^{(-1)}l^{(1)}-\omega^{(0)}l^{(0)})
\end{array}
\tag{27}
$$

The matrix elements of the operators (23) are now readily evaluated. Within the ground state representation one finds

$F_a^{1-}, F_{1,a}^{1-}$	$\rho^{(1)}$	$\rho^{(0)}$	π
$\omega^{(1)}$	0	0	$-\frac{1}{\sqrt{2}}$
$\omega^{(0)}$	0	0	0
η	$-\frac{1}{\sqrt{2}}$	0	0
$\rho^{(1)}$	1	$\frac{1}{\sqrt{2}}$	0
$\rho^{(0)}$	$\frac{1}{\sqrt{2}}$	1	0
π	0	0	1

$F_{5,a}^{1-}$	$\rho^{(1)}$	$\rho^{(0)}$	π
$\omega^{(1)}$	1	0	0
$\omega^{(0)}$	0	0	0
η	0	0	0
$\rho^{(1)}$	0	0	0
$\rho^{(0)}$	0	0	1
π	0	1	0

(28, 29)

Notice that F^{1-} and F_1^{1-} can be listed in the same table since the first contains only helicity non flip, the other only flip transitions. Only reduced matrix elements have been given. If X_a denotes an arbitrary isovector operator they are defined by

$$X^{11} \equiv \langle 1 \pm 1 | X_3 | 1 \pm 1 \rangle = \frac{1}{\sqrt{2}} \langle 10 | X_{\mp} | 1 \pm 1 \rangle ,$$

$$X^{10} \equiv \langle 1\; 0 | X_3 | 00 \rangle = \mp \frac{1}{\sqrt{2}} \langle 1 \pm 1 | X_{\pm} | 00 \rangle . \tag{30}$$

If we also introduce reduced matrix elements for isospin zero operators Y:

$$Y^{00} \equiv \langle 00 | Y | 00 \rangle , \quad Y^{11} \equiv \langle 1m | Y | 1m \rangle , \tag{31}$$

all operators (23) can be taken from (28), (29) and the following relations

$$[F_a^{1,\eta}]^{01} = [F_a^{1,-\eta}]^{11} = [F_0^{1,\eta}]^{00} = [F_0^{1,\eta}]^{11} , \tag{32}$$

which hold for $F^{1,\eta}$, $F_{1,2}^{1,\eta}$ and $F_5^{1,\eta}$. We see that due to the $(4, \overline{4})$ nature of the mesons, the $U(4) \times U(4)$ charges display full exchange degeneracy.

Using (28), (29) one can calculate the charges also for the $L = 1$ states (27), again showing exchange degeneracy (32). It can also easily be seen that if mesons consist of pure $(4, \bar{4})$ representations with arbitrary orbital excitations, relations (32) should hold for $F^{J\mp}(k)$ with any $J = 1, 2, 3, \ldots$ whatever the details of the model. Now, eq. (9) teaches us that trajectory functions of both signatures are equal and that all Regge residues satisfy the relations (32).

As we discussed in sect. 2, the matrix elements of $F_{5,a}^{1-}$ are related to the pionic couplings via the PCAC relation (14). In order to obtain decays of the $L = 1$ states to the ground states, mixing has to be introduced*. Since helicity and G parity are good quantum numbers, they have to be conserved in the mixing process. At $h = 0$ there is, in addition, normality (parity $\times\, e^{i\pi J_2}$) that must be respected. This leaves for isovector particles, only two allowed mixings: at $h = 0$

$$\pi = c\, \tilde{\pi} + s\, \tilde{A}_1 , \quad A_1 = -s\, \tilde{\pi} + c\, A_1 ; \tag{33}$$

and at $h = 1$

$$\rho = c'\, \tilde{\rho} + s'\, \tilde{B} , \quad B = -s'\, \tilde{\rho} + c'\, \tilde{B} , \tag{34}$$

where a tilde has been used to denote unmixed states and c, s, c', s' stand short for cos and sin of two different angles. The iso-singlet states η, D and ω, H can, in principle, mix independently. However, if we invoke SU(3), we expect the same mixing angles to appear. With these angles the charges (23) between $L = 0$ and $L = 1$ states

* Remember that J is not a good quantum number in the infinite momentum frame.

are listed in tables 1 and 2*. Also after mixing there is complete exchage degeneracy (32).

The matrix elements of $F^{1-}_{5,a}$ can be compared with the pionic decay widths. Good agreement with the many experimental numbers is found if one chooses the two parameters** to be $s \approx 0.8$, $s' \approx 0.41$ (i.e., $c \approx 0.6$, $c' \approx 0.91$). We recall one of the successes of the scheme: $\mathrm{B} \to \omega\pi$ decays only in the $h = 1$ state (with a rate $\Gamma_{\mathrm{B}\omega\pi} \approx 570\, s'^2 \sim 96$ MeV (exp. 100 ± 20). For the detailed comparison with the data the reader is referred to the original work***.

Table 1
The reduced matrix elements of the isovector part of the axial charges $F^{1-}_5(0)$ are listed for the mixed $(4, \bar{4})$ $L = 0$ and $L = 1$ meson states

<table>
<tr><th>F^{1-}_{5a}</th><th>$\rho^{(1)}$</th><th>$\rho^{(0)}$</th><th>$\pi^{(0)}$</th><th>$A_2^{(2)}$</th><th>$A_2^{(1)}$</th><th>$A_1^{(1)}$</th><th>$B^{(1)}$</th><th>$A_2^{(0)}$</th><th>$A_1^{(0)}$</th><th>$B^{(0)}$</th><th>$A_0^{(0)}$</th></tr>
<tr><td>$\omega^{(1)}$</td><td>c'^2</td><td colspan="2">0</td><td>0</td><td>0</td><td>0</td><td>$-c's'$</td><td colspan="4">0</td></tr>
<tr><td>$\omega^{(0)}$</td><td rowspan="2">0</td><td colspan="2" rowspan="2">0</td><td rowspan="2">0</td><td colspan="3" rowspan="2">0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>$\eta^{(0)}$</td><td>$\frac{s}{\sqrt{3}}$</td><td>0</td><td>0</td><td>$\sqrt{\frac{2}{3}}s$</td></tr>
<tr><td>$f^{(2)}$</td><td>0</td><td colspan="2">0</td><td>1</td><td colspan="3">0</td><td colspan="4">0</td></tr>
<tr><td>$f^{(1)}$</td><td>0</td><td colspan="2" rowspan="3">0</td><td rowspan="3">0</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td><td>0</td><td colspan="4" rowspan="3">0</td></tr>
<tr><td>$D^{(1)}$</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>$H^{(1)}$</td><td>$-s'c'$</td><td>0</td><td>0</td><td>s'^2</td></tr>
<tr><td>$f^{(0)}$</td><td rowspan="4">0</td><td>0</td><td>$\frac{s}{\sqrt{3}}$</td><td rowspan="4">0</td><td colspan="3" rowspan="4">0</td><td>0</td><td>$\frac{c}{\sqrt{3}}$</td><td>0</td><td>0</td></tr>
<tr><td>$D^{(0)}$</td><td>0</td><td>0</td><td>$\frac{c}{\sqrt{3}}$</td><td>0</td><td>0</td><td>$\sqrt{\frac{2}{3}}c$</td></tr>
<tr><td>$H^{(0)}$</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>$\sigma^{(0)}$</td><td>0</td><td>$\sqrt{\frac{2}{3}}s$</td><td>0</td><td>$\sqrt{\frac{2}{3}}c$</td><td>0</td><td>0</td></tr>
<tr><td>$\rho^{(1)}$</td><td>0</td><td colspan="2">0</td><td>0</td><td>$\frac{s'}{\sqrt{2}}$</td><td>$-\frac{s'}{\sqrt{2}}$</td><td>0</td><td colspan="4">0</td></tr>
<tr><td>$\rho^{(0)}$</td><td rowspan="2">0</td><td>0</td><td>c</td><td rowspan="2">0</td><td colspan="3" rowspan="2">0</td><td>0</td><td>$-s$</td><td>0</td><td>0</td></tr>
<tr><td>$\pi^{(0)}$</td><td>c</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>$A_2^{(2)}$</td><td>0</td><td colspan="2">0</td><td>0</td><td colspan="3">0</td><td colspan="4">0</td></tr>
<tr><td>$A_2^{(1)}$</td><td>$-\frac{s'}{\sqrt{2}}$</td><td colspan="2" rowspan="3">0</td><td rowspan="3">0</td><td>0</td><td>0</td><td>$\frac{c'}{\sqrt{2}}$</td><td colspan="4" rowspan="3">0</td></tr>
<tr><td>$A_1^{(1)}$</td><td>$\frac{s'}{\sqrt{2}}$</td><td>0</td><td>0</td><td>$-\frac{c'}{\sqrt{2}}$</td></tr>
<tr><td>$B^{(1)}$</td><td>0</td><td>$\frac{c'}{\sqrt{2}}$</td><td>$-\frac{c'}{\sqrt{2}}$</td><td>0</td></tr>
<tr><td>$A_2^{(0)}$</td><td rowspan="4">0</td><td>0</td><td>0</td><td rowspan="4">0</td><td colspan="3" rowspan="4">0</td><td>0</td><td>0</td><td>$\sqrt{\frac{2}{3}}$</td><td>0</td></tr>
<tr><td>$A_1^{(0)}$</td><td>$-s$</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>$B^{(0)}$</td><td>0</td><td>0</td><td>$\sqrt{\frac{2}{3}}$</td><td>0</td><td>0</td><td>$-\frac{1}{\sqrt{3}}$</td></tr>
<tr><td>$A_0^{(0)}$</td><td>0</td><td>0</td><td>0</td><td>0</td><td>$-\frac{1}{\sqrt{3}}$</td><td>0</td></tr>
</table>

Superscripts denote helicity. The exchange degenerate partner $F^{1+}_5(0)$ is obtained by using relation (32) of the text. The matrix elements of the isosinglet operator follows also from (32).

* The results can be checked by using the Adler-Weisberger type of sum rules for the reduced matrix elements implied by the commutation rules of the group. See the appendix.

** The angle $s \approx 0.8$ leads to $\Gamma_{\rho\pi\pi} \approx 270\, c^2$ MeV ~ 100 MeV.

*** See also the third of ref. [7] for a detailed comparison of the other couplings with experiment.

As we have argued in sect. 2, the pionic couplings *via* PCAC (15) can also be identified with the couplings of the π trajectory. Since the pion does not reggeize much, and the pionic decay widths do agree with experiments, the couplings of the π trajectory will be correct to the same degree of accuracy. Similarly, the exchange degenerate partner of the PCAC relation (14):

$$\frac{i}{f_\pi}(m_\beta{}^2 - m_\alpha{}^2)\,(F_5^{1+}(0))_{\beta\alpha} = (R^B)_{\beta\alpha}\Big|_{q^2=0}\,, \tag{35}$$

will yield correct couplings for the B trajectory as far as exchange degeneracy is a good approximation.

Table 2
The reduced matrix elements of the isovector charges $F^{1-}(0)$ and $F_1^{1-}(0)$ are displayed

$F_a^{1-}F_{1a}^{1-}$	$\rho^{(1)}$	$\rho^{(0)}$	$\pi^{(0)}$	$A_2^{(2)}$	$A_2^{(1)}$	$A_1^{(1)}$	$B^{(1)}$	$A_2^{(0)}$	$A_1^{(0)}$	$B^{(0)}$	$A_0^{(0)}$
$\omega^{(1)}$	0	0	$-\frac{c'c}{\sqrt{2}}-\frac{s's}{2}$	0		0		$\frac{s'}{2\sqrt{3}}$	$-\frac{s'}{2}$	0	$\frac{s'}{\sqrt{6}}$
$\omega^{(0)}$	0				0	0					
$\eta^{(0)}$	$-\frac{c'c}{\sqrt{2}}-\frac{s's}{2}$	0			0		$-\frac{sc'}{2}+\frac{cs'}{\sqrt{2}}$		0		
$f^{(2)}$	0	0		0		0			0		
$f^{(1)}$		0	0	0				0	0	$-\frac{1}{2}$	0
$D^{(1)}$	0	0	0	0		0		0	0	$-\frac{1}{2}$	0
$H^{(1)}$		0	$\frac{s'c}{\sqrt{2}}-\frac{c's}{2}$	$-\frac{1}{\sqrt{2}}$				$\frac{c'}{2\sqrt{3}}$	$-\frac{c'c}{2}-\frac{s's}{\sqrt{2}}$	0	$\frac{c'}{\sqrt{6}}$
$f^{(0)}$					0	0	$\frac{c'}{2\sqrt{3}}$				
$D^{(0)}$					0	0	$-\frac{cc'}{2}-\frac{ss'}{\sqrt{2}}$				
$H^{(0)}$	0	0		0	$-\frac{1}{2}$	$-\frac{1}{2}$	0		0		
$\sigma^{(0)}$					0	0	$\frac{c'}{\sqrt{6}}$				
$\rho^{(1)}$	1	$\frac{c'}{\sqrt{2}}$	0	0		0			0		
$\rho^{(0)}$	$\frac{c'}{\sqrt{2}}$	1	0	0	0	0	$-\frac{s'}{\sqrt{2}}$		0		
$\pi^{(0)}$	0	0	1		$-\frac{s}{2\sqrt{2}}$	$\frac{s}{2\sqrt{2}}$	0				
$A_2^{(2)}$	0	0		1	$\frac{1}{2}$	$-\frac{1}{2}$	0		0		
$A_2^{(1)}$		0 0	$-\frac{s}{2\sqrt{2}}$	$\frac{1}{2}$	1	0	0	$\sqrt{\frac{3}{8}}$	$-\frac{c}{2\sqrt{2}}$	0	0
$A_1^{(1)}$	0	0 0	$\frac{s}{2\sqrt{2}}$	$-\frac{1}{2}$	0	1	0	$\frac{1}{2\sqrt{6}}$	$\frac{c}{2\sqrt{2}}$	0	$-\frac{1}{\sqrt{3}}$
$B^{(1)}$		$-\frac{s'}{\sqrt{2}}$ 0	0	0	0	0	1	0	0	0	0
$A_2^{(0)}$					$\sqrt{\frac{3}{8}}$	$\frac{1}{2\sqrt{6}}$	0	1	0	0	0
$A_1^{(0)}$	0	0		0	$-\frac{c}{2\sqrt{2}}$	$\frac{c}{2\sqrt{2}}$	0	0	1	0	0
$B^{(0)}$					0	0	0	0	0	1	0
$A_0^{(0)}$					0	$-\frac{1}{\sqrt{3}}$	0	0	0	0	1

The first charge is helicity conserving and appears only in diagonal boxes. The second can flip the helicity indices up or down by one unit. To lowest order, these matrix elements coincide with the couplings of the ρ trajectory. The other partners ω, A_2, f, follow from exchange degeneracy (32).

Let us now test the Regge couplings of vector and tensor exchanges. A complete listing is given in table 2. Experimental tests are available only through πN scattering. From the diagonal elements of table 2 we see that with no helicity flip, π can only go into itself *via* ρ exchange or into η *via* A_2 exchange. Other resonances can only be produced at helicity ($\pm$ 1). Due to exchange degeneracy, the corresponding couplings can all be written as:

$$
\begin{aligned}
&\pi \to \rho^{(1)}, \omega^{(1)}: && -\frac{c'c}{\sqrt{2}} - \frac{s's}{2} \ (|\text{exp.}| \approx 20 \times \text{const}) \,, \\
&\pi \to A_2^{(2)}, f^{(1)}: && -\frac{s}{2\sqrt{2}} \qquad (|\text{exp.}| \approx 11 \times \text{const}) \,, \\
&\pi \to A_1^{(1)}, D^{(1)}: && \frac{s}{2\sqrt{2}} \qquad (|\text{exp.}| \approx 9 \times \text{const}) \,, \\
&\pi \to B^{(1)}, H^{(1)}: && \frac{s's}{\sqrt{2}} - \frac{c's}{2} \ (|\text{exp.}| \approx 11 \times \text{const}) \,,
\end{aligned}
\tag{36}
$$

for any of the allowed exchanges. For example: $\pi \to \omega^{(1)}$ can go *via* ρ or f exchange, $\pi \to \rho^{(1)}$ *via* A_2 or ω exchange, each of these couplings being $\sqrt{\frac{1}{2}}c'c - \frac{1}{2}s's$. The phenomenological determination of these couplings* has been given in parentheses in arbitrary units.

With the mixing angles $s = 0.8, c = 0.6; s' \sim 0.41, c' \sim 0.91$ we predict the couplings $-0.55, -0.28, +0.28, -0.2$, respectively. We see that apart from the unknown overall normalization there is good agreement with the experimental numbers. The B coupling comes out a little too small.

5. The baryon scheme

The mixing procedure for baryons is much more involved technically and more space will be needed for a full discussion. Here we just want to mention a few introductory points in order to complete the picture.

Baryons are assigned to (56,1) $L = 0$, (70,1) $L = 1$, (56,1) $L = 0{,}2$ etc, representation of U(6) $\times$ U(6). Then all Regge couplings are exchange degenerate according to (22).

The pionic couplings are known to emerge in good agreement with experiment [4, 5]. In addition we now find the results that $\rho^{(0)}, \omega^{(0)}$ (= $A_2^{(0)}, f^{(0)}$) non-flip trajectories can never excite the nucleon**.

At the level of no mixing, the matrix elements of $F^{1\mp}(0)$ and $F_1^{1\mp}(0)$ are listed for the (56,1) ground state in table 3. We see that the helicity flip couplings $\rho^{(1)}, \omega^{(1)}$ (= $A_2^{(1)}, f^{(1)}$) connect N and Δ in a pure M_1 transition and give elastic matrix ele-

* I am indebted to G. Fox for providing me with a ready-for-use version of the results of his paper.

** This statement holds in the approximation (14).

ments an F/D ratio of $\frac{2}{3}$, just as in the standard quark model*. Both results are in good agreement with experiment (13). Mixing brings a modification of these results by $\approx 30\%$. In addition, it causes transition to the $(70, L = 1)$ and higher states.

The ratios of N*, Δ* production with respect to NN or NΔ transitions are now provided together with their D/F ratios.

The couplings of $\rho^{(1)}$ to baryons can also be compared with the mesonic ones obtained previously. The agreements of NN and NΔ transitions is quite good (as is known from quark model estimates). For the excitation of N* resonances no experimental numbers are as yet available. Detailed phenomenological analyses are definitely needed.

Table 3
The reduced matrix elements of $F^{1^-}(0)$, $F_1^{1^-}(0)$ are shown for the (56,1) representation

$F^{1\mp}F_1^{1\mp}$	$(10,1)^{\frac{3}{2}}$	$(6,3)_{10}^{\frac{1}{2}}$	$(6,3)_8^{\frac{1}{2}}$	$(3,6)_8^{-\frac{1}{2}}$	$(3,6)_{10}^{-\frac{1}{2}}$	$(1,10)^{-\frac{3}{2}}$
$(10,1)^{\frac{3}{2}}$	1	$\frac{1}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$	0		0
$(6,3)_{10}^{\frac{1}{2}}$	$\frac{1}{\sqrt{3}}$	1	0	$-\frac{2}{3}$	$-\frac{2}{3}$	0
$(6,3)_8^{\frac{1}{2}}$	$-\frac{2}{\sqrt{3}}$	0	$\hat{F}$	$\frac{2}{3}\hat{F}+\hat{D}$	$-\frac{2}{3}$	0
$(3,6)_8^{-\frac{1}{2}}$	0	$-\frac{2}{3}$	$\frac{2}{3}\hat{F}+\hat{D}$	1	0	$\frac{2}{\sqrt{3}}$
$(3,6)_{10}^{-\frac{1}{2}}$		$-\frac{2}{3}$	$-\frac{2}{3}$	0	$\hat{F}$	$-\frac{1}{\sqrt{3}}$
$(1,10)^{-\frac{3}{2}}$	0	0		$\frac{2}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	1

The reduced matrix elements are defined as $2\langle\Delta^+|F_3^{1^-}|\Delta^+\rangle$, $\sqrt{2}\langle\Delta^+|F_3^{1^-}|p\rangle$ and $\langle p|F_3^{1^-}|p\rangle$ with $\langle p|F|p\rangle = \langle p|D|p\rangle = \frac{1}{2}$. To lowest order, they coincide with the couplings of ρ and A_2 trajectory. Rows and columns have been labelled directly in terms of the chiral properties of the states ρ and Δ at different helicities. This saves one from writing down $F_5^{1\mp}$ explicity. .

6. Conclusions

The new algebra of signatured form factors is seen to provide a promising tool for the understanding of Regge couplings in terms of quarks. In this work we have defined the algebraic structure, formulated a program, and given a simple illustra-

* Note that the matrix elements of $F_1^{1^-}(0)$ have to be interpreted as the helicity flip contribution of the total moment coupling of the ρ trajectory. This has been one of the important results of the discussion of Melosh [5]. The reason is that $\hat{F}_1^{1^-}(0)$ commutes with $P^\perp$, a property which is shared by the $(1 + \kappa)$ type of coupling, not with κ at infinite momentum. This is analogous to the electromagnetic case where $D_\perp = \int d^2\, x d\bar{x}\, x_\perp j^+{}_{em}$ does not commute with $P^\perp$ but $M_\perp = P^+D_\perp + E_\perp Q$ does.

tion to some lowest approximation. There are many questions which have to be answered by future investigations.

(i) What are the full implications of the algebra and its representations upon the Pomeranchuk singularity? It appears as if s channel helicity conservation will appear as a lowest order results due to the U(6) × U(6) singlet property.

(ii) How does one extend the representation of $F_a^{J\mp}(0)$ properly to all J? Then also the electromagnetic and weak structure functions are predicted and can be compared with experiment. It is quite probable that here we will run into difficulties with the usual quark wave functions. From our experience with dual models it appears that wave functions with one principal and one orbital quantum number are not enough to allow for the full hadronic singularity structure in the J plane. The richer Veneziano type of spectrum will be necessary in order to solve this problem. But then there are problems in extending the algebra to all k. Until today nobody knows how to obtain even the electromagnetic form factors in the Veneziano model.

(iii) What are the connections with the auxiliary Reggeon fields of the Gribov calculus [1]? Is there an approximate phenomenological field theory involving directly the operator $\hat{F}^{J\mp}(k)$? Can it be derived from the quark-gluon model? A Langrangian with U(6) × U(6) symmetry incorporating the field current identity at infinite momentum and having an additional space-time variable conjugate* to $\omega = 1-J$ provides certainly an interesting example. It represents a solution of our full algebra (10) with the correct leading Regge trajectories appearing to lowest order in perturbation theory and higher order graphs giving unitarity corrections. Independent of any definite model, our algebra gives important restrictions on how to incorporate internal degrees of freedom with Gribov's fields. It may also point the way of how to accommodate particles in this hitherto purely reggeonic theory.

I am grateful to Murray Gell-Mann for his kind hospitality at Caltech. It is a pleasure to thank him and Richard Feynman for generating an extremely stimulating atmosphere at their institute. There are many things I have learned in discussions with both of them, with Harald Fritsch, Heinrich Leutwyler and Peter Minkowski. There were also some clarifying conversations with Henry Abarbanel, Richard Brandt, Hugh Osborn and Yüval Ne'eman.

My special thanks go to Geoffrey C. Fox. Not only did he stress the importance of understanding exchange degeneracy, he also kept me informed about the phenomenological aspects of Regge couplings and provided me with the experimental results quoted in the text.

Appendix. Adler-Weisberger type of sum rules for reduced matrix elements

The results of our calculations presented in tables 1 and 2 have to satisfy certain

* By virtue of eq. (6) it would be related to ξ as $\tau = i \log \xi$.

sum rules which are the analogue of the Adler-Weisberger sum rules of $SU(3) \times SU(3)$. They arise by eliminating the isospin indices from the commutator of the group and going to reduced matrix elements defined in (30), (31). These sum rules are useful in checking final results when representing any group containing isospin.

Consider the commutators

$$[A_a, B_b] = i\epsilon_{abc}\, C_c\,, \tag{A.1}$$

$$[A_a, B_b] = i\delta_{ab}\, C\,, \tag{A.2}$$

$$[A_a, B] = i\, C_b\,. \tag{A.3}$$

The commutators (A.1) translate to reduced matrix elements as

$$A^{01}B^{10} - B^{01}A^{10} = 0\,, \quad A^{01}B^{11} + B^{01}A^{11} = C^{01}\,,$$

$$A^{10}B^{01} + B^{11}A^{11} = C^{11}\,, \quad A^{11}B^{11} - B^{11}A^{11} = 0\,. \tag{A.1$'$}$$

In the case of (A.2) one has to substitute the right-hand sides by iC^{00}, 0, 0, iC^{11}, respectively. For the third commutator, finally, one finds

$$A^{01}B^{11} - B^{00}A^{10} = iC^{01}\,, \quad A^{11}B^{11} - B^{11}A^{11} = iC^{11}\,. \tag{A.3$'$}$$

The original Adler-Weisberger relations are obtained from the commutator

$$[X_a X_b] = i\epsilon_{abc}\, T_c\,,$$

where X are the matrix elements F_5^{1-} and T is the isospin. Using (A.1$'$) yields the well-known sum rules:

$$X^{01}X^{11} = 0\,, \quad X^{10}X^{01} + X^{11}X^{11} = 1\,. \tag{A.4}$$

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