DECAY RATES OF THE DECUPLET RESONANCES FROM O(3, 1)⊗SU(3) DYNAMICS. DEFINITE SU(3) SYMMETRY BREAKING*

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The simplest dynamical group including SU(3), O(3, 1)⊗ SU(3), is used to describe the decay widths of the $j^P = \frac{3}{2}^+$ decuplet resonances. SU(3) breaking comes in naturally (and uniquely) through the mass differences in the multiplet. The only two parameters of the theory have been fixed in an independent fit of the $I=\frac{3}{2}$ baryon decay rates for different spins. The agreement with experiment is excellent.

Recently, the idea of a dynamical group governing baryon form factors has passed an impressive test. The simplest possible dynamical group, O(3, 1), has been able to describe the pionic decay rates of many resonances extremely well using only two parameters, a coupling constant G and a Casimir operator ν specifying the representation.[1]

We recall that after extension by parity and assuming the existence of an electromagnetic current operator on the representation space, the fermion spectrum consists of a tower of particles with spins $\frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \cdots$ or $\frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \cdots$ and their antiparticles. Neglecting isospin, the nucleon and the isospin-$\frac{1}{2}$ resonances $N^*(1525)$, $N^*(1688)$, $N^*(2190)$, $N^*(2650)$, and $N^*(3030)$ are assigned to the first representation and the isospin-$\frac{3}{2}$ resonances $\Delta(1236)$, $\Delta(1920)$, $\Delta(2420)$, $\Delta(2850)$, and $\Delta(3230)$ to the second one, both with the same Casimir operator ν. The process of a baryon decaying into a lower state under emission of a pion is then described in the following way: One uses the special frame where the initial baryon $|a',j'\rangle$ is at rest and the final one $|a,j\rangle$ moves with rapidity ζ $[\zeta = \tanh^{-1}(v/c)]$ into the z direction and interprets the matrix element

$$A_m^{j'j}(\zeta) = (G/\sqrt{2})\langle a'j'm|Pe^{i\vec{M}\cdot\vec{\zeta}}|ajm\rangle \qquad (1)$$

as the transition amplitude. G is a coupling constant for the whole multiplet, $\vec{M}$ denotes the Lorentz generators of O(3, 1), and P is the only pseudoscalar in the representation space, $P = \vec{L}\cdot\vec{M}/|\vec{L}\cdot\vec{M}|$. This interpretation is suggested by the observation that hydrogenic electric form factors can be written in the same form.[2-4] $A_m^{j'j}(\zeta)$ can be given in terms of global representations of O(3, 1) as

$$A_m^{j'j}(\zeta) = (G/\sqrt{2})B_m^{\pm j'j}(\zeta[\tfrac{1}{2},\nu]) \qquad (2)$$

with

$$B_m^{\pm j'j}(\zeta[\tfrac{1}{2},\nu]) = \tfrac{1}{2}\{B_m^{j'j}(\zeta[\tfrac{1}{2},\nu]) \pm B_m^{j'j}(\zeta[\tfrac{1}{2},\nu])\}, \qquad (3)$$

where ν is the only undetermined Casimir operator of O(3, 1) and + or − have to be used according to whether the parities of the ground states of the initial and final baryon multiplets are opposite or equal. $B_m^{j'j}$ is given explicitly in terms of hypergeometric functions by[5]

$$B_j^{j'j}(\zeta[\tfrac{1}{2},\nu]) = N^{j'j}\sinh^{j'-j}\zeta\, e^{i\nu\zeta} e^{-(j'+\frac{3}{2})\zeta} F(j'+1-i\nu, j'+\tfrac{3}{2}, 2j'+2, 1-e^{-2\zeta}),$$

$$N^{j'j} = 2^{j'-j}\left[\frac{(j'+\frac{1}{2})!(j'-\frac{1}{2})![(j+1)^2+\nu^2]\cdots(j'^2+\nu^2)(2j+1)!(j'+j)!}{(j+\frac{1}{2})!(j-\frac{1}{2})!(2j')!(2j'+1)!(j'-j)!}\right]^{1/2}. \quad (4)$$

The decay rates for the Δ resonances decaying into $N+\pi$ can then be calculated from

$$\Gamma = \varphi G^2 |B_{1/2}^{+j'\frac{1}{2}}(\zeta[\tfrac{1}{2},\nu])|, \quad (5)$$

where φ is the invariant phase space

$$\varphi = [1/(2j'+1)](p/M_i)M_f\,\mu \quad (6)$$

with obvious notation. Using $G = 12.5\ \mathrm{BeV}^{-1/2}$ and $\nu = 3.5$, Γ fits the experimental decay rates extremely well.

The purpose of this note is to point out, that one can include broken SU(3) into the same theory (i.e., using the same G and ν) by simply extending the dynamical group to O(3, 1)⊗SU(3).[6] We assume that the O(3, 1) multiplet containing the nucleon is a tower of SU(3) octets while all the resonances lie in a decuplet tower. We then predict the decay rates of all $j=\frac{3}{2}^+$ decuplet members.

This is done in the following way: Instead of the pseudoscalar operator P in the expression for the decay amplitude [Eq. (1)] we choose $P\times\lambda^i$ to represent the pseudoscalar meson octet, where λ^i are the eight generators of SU(3). The Lorentz generators M remain the same. Normalizing the SU(3) Clebsch-Gordan coefficients (CG) to be unity for $\Delta\to N+\pi$, the decay rates become then

$$\Gamma = \varphi G^2(\mathrm{CG})^2 |B_{1/2}^{+j'\frac{1}{2}}(\zeta[\tfrac{1}{2}, 3.5])|^2 \quad (7)$$

with $j'=\frac{3}{2}$ for the $\frac{3}{2}^+$ decuplet. To compare the theory with experiment we have used this equation to calculate the matrix element B^+ from the observed decay rates and plotted them onto a three-dimensional graph of the theoretical $B_{1/2}^{+j'\frac{1}{2}}(\zeta[\frac{1}{2}, 3.5])$ as a function of the spin of the decaying particle j' and the variable $p/M_f = \sinh\zeta$. For completeness we have also inserted the experimental B values for the decay series $\Delta\to N+\pi$ which were used to adjust the parameters G and ν. We see that the agreement is excellent for all points (see Fig. 1).

Observe that SU(3) is broken in this approach in a definite way, once the particle masses are given. First, the phase space is clearly not invariant under SU(3) due to the mass differences in one multiplet. Second, the decay amplitude is a product of an SU(3) Clebsch-Gordan coefficient and a universal function of the rapidity ζ. Different members of an SU(3) multiplet decay with different rapidities due to the mass differences and this breaks SU(3) symmetry in the amplitudes.

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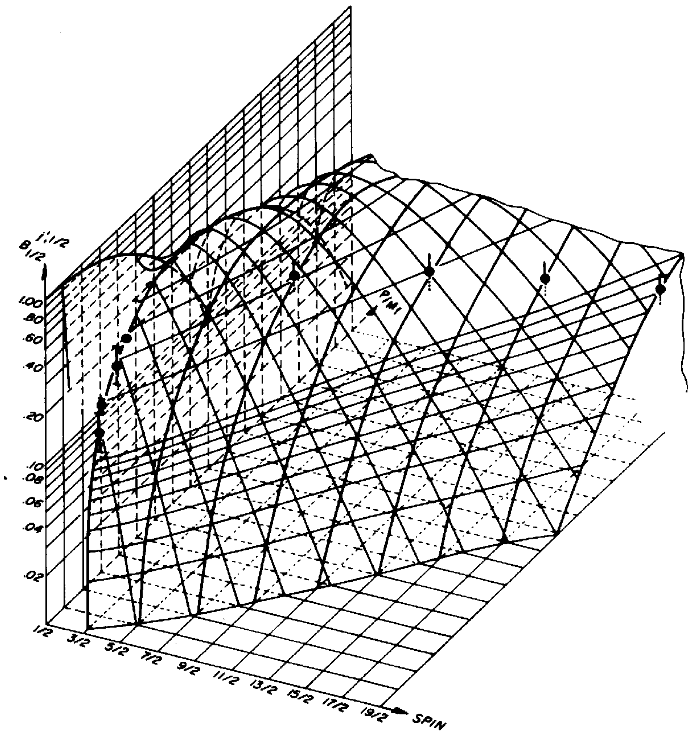

FIG. 1. The experimental decay amplitudes calculated from Eq. (5) are plotted and compared with the amplitude of the theory $B_{1/2}^{j'\frac{1}{2}}$ as a function of the spin of the decaying particles and the variable P/M_f. Counted in a clockwise direction, the dots stand for the decay amplitudes of $\Sigma(1385)\to\Sigma+\pi$, $\Xi(1530)\to\Xi+\pi$, $\Sigma(1385)\to\Lambda+\pi$, $\Delta(1236)\to N+\pi$, $\Delta(1920)\to N+\pi$, $\Delta(2420)\to N+\pi$, $\Delta(2850)\to N+\pi$, $\Delta(3230)\to N+\pi$. For the resonances with spin larger than $\frac{3}{2}$ the experimental errors are not known.

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